# LESSON 7.5 Operations with Matrices

## Representation of Matrices

- A matrix can be denoted by an uppercase letter such as A, B, or C.
- A representative element enclosed in brackets, such as $\left[a_{ij}\right]$.
- A rectangular array of numbers like the ones we have seen.

## Matrix Addition and Scalar Multiplication

### Matrix Addition

- Let A and B be matrices of order mXn, then A+B= $\left[a_{ij}+b_{ij}\right]$
- **When adding matrices, add the corresponding elements.**
- The sum of two matrices of different orders is undefined.

### Scalar Multiplication

- Let A be a matrix of order mXn and c be a scalar, then cA= $\left[ca_{ij}\right]$
- **When multiplying a matrix by a scalar you multiply each element by c.**

# Properties of Matrix Addition and Scalar Multiplication

## Properties of Matrix Addition and Scalar Multiplication

Let $A$, $B$, and $C$ be $m \times n$ matrices and let $c$ and $d$ be scalars.

1. $A + B = B + A$ Commutative Property of Matrix Addition
2. $A + (B + C) = (A + B) + C$ Associative Property of Matrix Addition
3. $(cd)A = c(dA)$ Associative Property of Scalar Multiplication
4. $1A = A$ Scalar Identity
5. $A + O = A$ Additive Identity
6. $c(A + B) = cA + cB$ Distributive Property
7. $(c + d)A = cA + dA$ Distributive Property

Example Addition of Matrices

$$1.\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$

$$2.\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

3. The sum of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}$$

is undefined because A is of order 2X3 and B is of order 2X2.

Example

## Scalar Multiplication

For the following matrices, find (a) -B and (b) A-B.

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix}$$ and

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

Example

Using the Distributive Property

$$3\left(\begin{bmatrix}-2 & 0\\ 4 & 1\end{bmatrix}+\begin{bmatrix}4 & -2\\ 3 & 7\end{bmatrix}\right)$$

c) Find the total number of lbs for SS and Al.

**83. Computing the Cost of Production** The Acme Steel Company is a producer of stainless steel and aluminum containers. On a certain day, the following stainless steel containers were manufactured: 500 with 10-gallon (gal) capacity, 350 with 5-gal capacity, and 400 with 1-gal capacity. On the same day, the following aluminum containers were manufactured: 700 with 10-gal capacity, 500 with 5-gal capacity, and 850 with 1-gal capacity.

(a) Find a 2 by 3 matrix representing these data. Find a 3 by 2 matrix to represent the same data.

(b) If the amount of material used in the 10-gal containers is 15 pounds (lb), the amount used in the 5-gal containers is 8 lb, and the amount used in the 1-gal containers is 3 lb, find a 3 by 1 matrix representing the amount of material used.

(c) Multiply the 2 by 3 matrix found in part (a) and the 3 by 1 matrix found in part (b) to get a 2 by 1 matrix showing the day's usage of material.

(d) If stainless steel costs Acme \$0.10 lb and aluminum costs \$0.05 lb, find a 1 by 2 matrix representing cost.

(e) Multiply the matrices found in parts (c) and (d) to determine the total cost of the day's production.

S.S. Al.

$.1 \cdot 11500 = \$1150$

$.05 \cdot 17050 = \$852.50$

$\$\, 1150 + 852.50$

$$\begin{bmatrix} .1 & .05 \end{bmatrix} \begin{bmatrix} 11500 \\ 17050 \end{bmatrix} = \begin{bmatrix} 2002.50 \end{bmatrix}$$

$1 \times 2 = 2 \times 1$

# LESSON 7.5 Day 2

## Matrix Multiplication

- Let A be a matrix of order mXn, and B be a matrix of order nXp, the

  AB= $\left[c_{ij}\right]$

  where $\left[c_{ij}\right] = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$

- When multiplying matrices, multiply the corresponding elements in each row of A with each column of B and add them.

## Requirement to Multiply Matrices

A x B = AB

m x n     n x p     m x p

Equal

Order of AB

# Properties of Matrices

$IA = A$

**Properties of Matrix Multiplication**

Let $A$, $B$, and $C$ be matrices and let $c$ be a scalar.

1. $A(BC) = (AB)C$ — Associative Property of Matrix Multiplication
2. $A(B + C) = AB + AC$ — Left Distributive Property
3. $(A + B)C = AC + BC$ — Right Distributive Property
4. $c(AB) = (cA)B = A(cB)$ — Associative Property of Scalar Multiplication

$2 \times (3 \times 4)$

# Definition of Identity Matrix

$I_{3\times 3}$

$I_{2\times 2}$

**Definition of Identity Matrix**

The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere is called the **identity matrix of order $n$** and is denoted by

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}. \quad \text{Identity matrix}$$

Note that an identity matrix must be *square*. When the order is understood to be $n$, you can denote $I_n$ simply by $I$.

Example

Matrix Multiplication

1. $\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

1 x 3 3 x 1

$1 \cdot 2 + -2 \cdot -1 + -3 \cdot 1$

$2 + 2 - 3$

$\begin{bmatrix} 1 \end{bmatrix}$

$a_{nm}$

2. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix} = A$

3 x 1 1 x 3

Example

Solving a System of Linear Equations

$$\begin{cases} x_1 - 2x_2 + x_3 = -4 \\ x_2 + 2x_3 = 4 \\ 2x_1 + 3x_2 - 2x_3 = 2 \end{cases}$$

a. Write the system as a matrix equation AX=B.
b. Use Gauss-Jordan elimination on $[A \vdots B]$ to solve for matrix X.