LESSON 7.5 Operations with Matrices

Representation of Matrices

- A matrix can be denoted by anuppercase lettersuch as A, B, or C.
- A representative elementenclosed in brackets, such as a_{ij} .
- A rectangular array of numbers like the ones we have seen.

Matrix Addition and Scalar Multiplication Matrix Addition

- Let A and B be matrices of order mXn, then A+B= $\left[a_{ij}+b_{ij}\right]$
- When adding matrices, add the corresponding elements.
- The sum of two matrices of different orders is undefined.

Scalar Multiplication

- ullet Let A be a matrix of order mXn and c be a scalar, then cA= $\left[ca_{ij} \right]$
- When multiplying a matrix by a scalar you multiply each element by c.

Properties of Matrix Addition and Scalar Multiplication

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ matrices and let c and d be scalars.

1.
$$A + B = B + A$$

Commutative Property of Matrix Addition

2.
$$A + (B + C) = (A + B) + C$$

Associative Property of Matrix Addition

$$3. (cd)A = c(dA)$$

Associative Property of Scalar Multiplication

4.
$$1A = A$$

Scalar Identity

5.
$$A + O = A$$

Additive Identity

6.
$$c(A + B) = cA + cB$$

Distributive Property

7.
$$(c+d)A = cA + dA$$

Distributive Property

Example Addition of Matrices

$$1.\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$
$$2.\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

3. The sum of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}$$

is undefined because A is of order 2X3 and B is of order 2X2.

Scalar Multiplication

For the following matrices, find (a) -B and (b) A-B.

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \quad \text{and} \quad$$

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

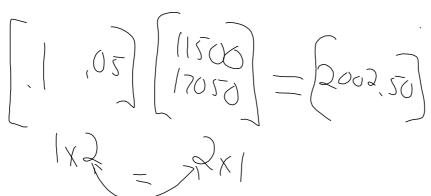
Using the Distributive Property

$$3\left(\begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 3 & 7 \end{bmatrix}\right)$$

Find the total number of lbs for

- pany is a producer of stainless steel and aluminum containers. On a certain day, the following stainless steel containers were manufactured: 500 with 10-gallon (gal) capacity, 350 with 5-gal capacity, and 400 with 1-gal capacity. On the same day, the following aluminum containers were manufactured: 700 with 10-gal capacity, 500 with 5-gal capacity, and 850 with 1-gal capacity.
 - (a) Find a 2 by 3 matrix representing these data. Find a 3 by 2 matrix to represent the same data.
 - (b) If the amount of material used in the 10-gal containers is 15 pounds (lb), the amount used in the 5-gal containers is 8 lb, and the amount used in the 1-gal containers is 3 lb, find a 3 by 1 matrix representing the amount of material used.
 - (c) Multiply the 2 by 3 matrix found in part (a) and the 3 by 1 matrix found in part (b) to get a 2 by 1 matrix showing the day's usage of material.
 - (d) If stainless steel costs Acme \$0.10 lb and aluminum costs \$0.05 lb, find a 1 by 2 matrix representing cost.
 - (e) Multiply the matrices found in parts (c) and (d) to determine the total cost of the day's production.

 $0.|\cdot||500 = ||1|50$ $0.|\cdot||500 = ||4||50$ $0.|\cdot||500 = ||4||50$ $0.|\cdot||500 = ||4||50$

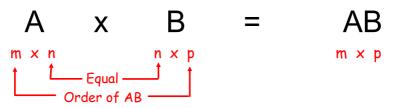


LESSON 7.5 Day 2

Matrix Multiplication

- Let A be a matrix of order mXn, and B be a matrix of order nXp, the $AB = \begin{bmatrix} c_{ij} \end{bmatrix}$ where $\begin{bmatrix} c_{ij} \end{bmatrix} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{in}b_{nj}$
- When multiplying matrices, multiply the corresponding elements in each row of A with each column of B and add them.

Requirement to Multiply Matrices



Properties of Matrices

Properties of Matrix Multiplication

Let A, B, and C be matrices and let c be a scalar.

1.
$$A(BC) = (AB)C$$

Associative Property of Matrix Multiplication

$$2. A(B+C) = AB + AC$$

Left Distributive Property

$$3. (A+B)C = AC + BC$$

Right Distributive Property

4.
$$c(AB) = (cA)B = A(cB)$$

Associative Property of Scalar Multiplication



Definition of Identity Matrix

Definition of Identity Matrix

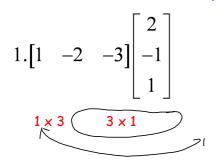
The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere is called the **identity matrix of order** n and is denoted by

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}.$$
 Identity matrix

Note that an identity matrix must be *square*. When the order is understood to be n, you can denote I_n simply by I.

$$T_{2x^2}$$

Matrix Multiplication



$$1.\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$1 \times 3 \quad 3 \times 1$$

$$2.\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \qquad \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & -3 \end{bmatrix}$$

Solving a System of Linear Equations

$$\begin{cases} x_1 - 2x_2 + x_3 = -4 \\ x_2 + 2x_3 = 4 \\ 2x_1 + 3x_2 - 2x_3 = 2 \end{cases}$$

- a. Write the system as a matrix equation AX=B. b. Use Gauss-Jordan elimination on $\begin{bmatrix} A \\ B \end{bmatrix}$ to solve for matrix X.