LESSON 7.6 Inverse of a Square Matrix

We can't divide by a matrix so we need a different way to solve matrix

equations.

Example /

 $A' \cdot AX = B$ $X = A' \cdot B$ $X = A' \cdot B$

Definition of the Inverse of a Square Matrix

Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that

$$AA^{-1} = I_n = A^{-1}A$$

then A^{-1} is called the inverse of A. The symbol A^{-1} is read "A inverse.

If a matrix has an inverse it is called invertible. A non-square matrix $A^{-1} A = Inxn = A \cdot A^{-1}$ can't have an inverse.

Example

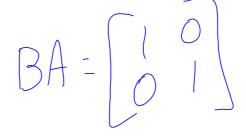
Show that A and B are inverses.

$$B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$$

Solution

Show that

$$\overrightarrow{AB} = \overrightarrow{I} \neq \overrightarrow{BA}$$



Example

If possible, find the inverse of the following matrix using your calculator.

$$C = \begin{bmatrix} 1 & \cancel{0} & 0 \\ \cancel{0} & \phi & \cancel{0} \\ \cancel{0} & \cancel{2} & -\cancel{3} \end{bmatrix} \begin{bmatrix} 1 & \cancel{0} & \cancel{0} \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 7 & -7 & -7 & -7 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Check-Point:

Verify that the following matrices are inverses.

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 0.75 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

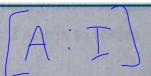
$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

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Finding an Inverse Matrix

Let A be a square matrix of order n.



- 1. Write the $n \times 2n$ matrix that consists of the given matrix A on the left and the $n \times n$ identity matrix I on the right to obtain [A : I].
- **2.** If possible, row reduce A to I using elementary row operations on the *entire* matrix [A : I]. The result will be the matrix $[I : A^{-1}]$. If this is not possible, A is not invertible.
- 3. Check your work by multiplying to see that $AA^{-1} = I = A^{-1}A$.

Inverse of a 2x2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Determinant

A is invertible if and only if $ad - bc \neq 0$.

The inverse is given by:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solving Systems Using Inverses

$$\begin{cases} x + 2y - z = 4 \\ -x - y + 2z = 16 \end{cases} \longrightarrow \begin{bmatrix} 1 & 2 & -1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 5 \end{bmatrix}$$
$$AX = B$$
$$X = A^{-1}B$$

A System of Equations with a Unique Solution

If A is an invertible matrix, the system of linear equations represented by AX = B has a unique solution given by

$$X = A^{-1}B.$$

To solve the system, multiply the inverse of matrix A with the constant matrix, B. $A^{-1}B=X$.