Calculus 7.2 Story Problems

(39) a) $\int R'(x)\,dx = R(x)$

$\int 4x(x^2+27000)^{-2/3}\,dx$

$u = x^2+27000$

$du = 2x\,dx$

$2du = 4x\,dx$

$\int 2u^{-2/3}\,du$

$= \frac{2u^{1/3}}{1/3} + C = 6(x^2+27000)^{1/3} + C$

$29.591 = 6(125^2+27000)^{1/3} + C$

$-180 = C$

$\boxed{R(x) = 6(x^2+27000)^{1/3} - 180}$

b)

$40 = 6(x^2+27000)^{1/3} - 180$

$220 = 6(x^2+27000)^{1/3}$

$36.666 = (x^2+27000)^{1/3}$

$49296.2963 = x^2+27000$

$22296.2963 = x^2$

$149.32 = x$ round up $\boxed{150}$

(40) a) $D(t) = \int D'(t)\,dt = \int 90(t+6)(t^2+12t)^{1/2}\,dt$

$u = t^2+12t$

$du = 2t+12\,dt$

$\frac{1}{2}du = (t+6)\,dt$

$45du = 90(t+6)\,dt$

$= \int 45u^{1/2}\,du$

$= \frac{45u^{3/2}}{3/2} + C = 30u^{3/2} + C$

$D(t) = 30(t^2+12t)^{3/2} + C$

$16260 = 30(4^2+12(4))^{3/2} + C$

$16260 = 15360 + C \rightarrow C = 900$

$\boxed{D(t) = 30(t^2+12t)^{3/2} + 900}$

(40b) $40000 = 30(t^2+12t)^{3/2} + 900$

$1303.33 = (t^2+12t)^{3/2}$

$119.31 = t^2+12t$

$0 = t^2+12t-119.31$ $\quad t = \dfrac{-12 \pm \sqrt{144-4(1)(-119.31)}}{2}$

$\boxed{t = 6.46}$ start of 7th year

(41) a) $C(x) = \int C'(x)\,dx = \int (60x)(5x^2+e)^{-1}\,dx$

$u = 5x^2+e$ $\quad = \int 6u^{-1}\,du$

$du = 10x\,dx$ $\quad = 6\ln|u| + C$

$6\,du = 60x\,dx$ $\quad C(x) = 6\ln|5x^2+e| + C$

$10 = 6\ln|e| + C$

$10 = 6 + C$

$4 = C$ $\quad \boxed{C(x) = 6\ln|5x^2+e| + 4}$

b) $C = 6\ln|125+e| + 4$

$C = 33098.96 > 20000$ $\quad$ Yes

(42) $P(x) = \int P'(x)\,dx = \int x e^{-x^2}\,dx$

$u = -x^2$ $\quad = \int -\frac{1}{2} e^u\,du$

$du = -2x\,dx$ $\quad = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$

$-\frac{1}{2}du = x\,dx$

$.01 = -\frac{1}{2}e^{-9} + C$

$.01006 = C$ $\quad \boxed{P(x) = -\frac{1}{2}e^{-x^2} + .01}$

b) $\lim_{x\to\infty} P(x) = \boxed{\#10000}$

(44) $F(t) = \int f'(t)\,dt = \int .001483t\,(t-1980)^{.75}\,dt$

$u = t - 1980$

$du = dt$

$t = u + 1980$

$.001483t = .001483u + 2.93634$

$\int (.001483u + 2.93634)\,u^{.75}\,du$

$= \int .001483u^{1.75} + 2.93634u^{.75}\,du$

$= \frac{.001483u^{2.75}}{2.75} + \frac{2.93634u^{1.75}}{1.75} + C$

$F(t) = .0005393(t-1980)^{2.75} + 1.6779(t-1980)^{1.75} + C$

$262.951 = 0 + 0 + C$

$F(t) = .0005393(t-1980)^{2.75} + 1.6779(t-1980)^{1.75} + 262.951$

$F(2012) = .0005393(32^{2.75}) + 1.6779(32^{1.75}) + 262.951$

$\approx 992{,}786{,}088$