4.2 Product Rule & Quotient Rule

Day 1 #1-10, 28, 37

① $\frac{dy}{dx} = 6x(2x-1) + (3x^2+2)(2) = 12x^2 - 6x + 6x^2 + 4$

$\boxed{\frac{dy}{dx} = 18x^2 - 6x + 4}$

② $\frac{dy}{dx} = 10x(4x+3) + (5x^2-1)(4) = 40x^2 + 30x + 20x^2 - 4$

$\boxed{\frac{dy}{dx} = 60x^2 + 30x - 4}$

③ $y = (2x-5)(2x-5)$

$\frac{dy}{dx} = 2(2x-5) + (2x-5)(2) = 4x-10 + 4x-10 = \boxed{8x-20}$

④ $y = (7x-6)(7x-6)$

$\frac{dy}{dx} = 7(7x-6) + (7x-6)(7) = 49x-42 + 49x-42 = \boxed{98x-84}$

⑤ $k(t) = (t^2-1)(t^2-1)$

$k'(t) = (2t)(t^2-1) + (t^2-1)(2t) = 2t^3 - 2t + 2t^3 - 2t = \boxed{4t^3 - 4t}$

⑥ $g(t) = (3t^2+2)(3t^2+2)$

$g'(t) = (6t)(3t^2+2) + (3t^2+2)(6t) = 18t^3 + 12t + 18t^3 + 12t$

$\boxed{g'(t) = 36t^3 + 24t}$

⑦ $y = (x+1)(x^{1/2}+2)$

$\frac{dy}{dx} = (1)(x^{1/2}+2) + (x+1)\left(\frac{1}{2\sqrt{x}}\right) = (\sqrt{x}+2) + \frac{x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \sqrt{x} + 2 + \frac{1}{2}\sqrt{x} + \frac{1}{2\sqrt{x}} = \boxed{\frac{3}{2}\sqrt{x} + 2 + \frac{1}{2\sqrt{x}}}$

(8) $y=(2x-3)(x^{\frac{1}{2}}-1)$

$\frac{dy}{dx} = 2(x^{\frac{1}{2}}-1) + (2x-3)\left(\frac{1}{2\sqrt{x}}\right) = 2x^{\frac{1}{2}} - 2 + \frac{2x}{2\sqrt{x}} - \frac{3}{2\sqrt{x}}$

$= 2\sqrt{x} - 2 + \sqrt{x} - \frac{3}{2\sqrt{x}} = \boxed{3\sqrt{x} - 2 - \frac{3}{2\sqrt{x}}}$

(9) $p(y) = (-1y^{-2} - 2y^{-3})(2y^{-3} - 5y^{-4}) + (y^{-1} + y^{-2})(-6y^{-4} + 20y^{-5})$

Foil → $= -2y^{-5} + 5y^{-6} - 4y^{-6} + 10y^{-7} + (-6y^{-5} + 20y^{-6} - 6y^{-6} + 20y^{-7})$

Like terms → $= -8y^{-5} + 15y^{-6} + 30y^{-7} \Rightarrow \boxed{p(y) = \frac{-8}{y^5} + \frac{15}{y^6} + \frac{30}{y^7}}$

(10) $g'(x) = (-2x^{-3} + 3x^{-4})(3x^{-1} + 4x^{-4}) + (x^{-2} - x^{-3})(-3x^{-2} - 16x^{-5})$

$= -6x^{-4} - 8x^{-7} + 9x^{-5} + 12x^{-8} + (-3x^{-4} - 16x^{-7} + 3x^{-5} + 16x^{-8})$

$= -9x^{-4} + 12x^{-5} - 24x^{-7} + 28x^{-8}$

$\boxed{g'(x) = \frac{-9}{x^4} + \frac{12}{x^5} - \frac{24}{x^7} + \frac{28}{x^8}}$

(28) $h(x) = f(x)g(x)$

$h'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$h'(3) = f'(3)\cdot g(3) + f(3)\cdot g'(3) = 8\cdot 4 + 9\cdot 5 = 32 + 45 = \boxed{77}$

(37) $f'(x) = 2x(x^2 - \sqrt{2}) + (x^2 - 2)(2x) = 2x^3 - 2x\sqrt{2} + 2x^3 - 4x$

$f'(x) = 4x^3 - 2x\sqrt{2} - 4x = 4x^3 - x(2\sqrt{2} + 4)$

$4x^3 - x(2\sqrt{2} + 4) = 0$

$x(4x^2 - (2\sqrt{2} + 4)) = 0$

$\boxed{x = 0}$

$4x^2 - (2\sqrt{2} + 4) = 0$

$4x^2 - 6.828 = 0$

$\frac{4x^2}{4} = \frac{6.828}{4}$

$\sqrt{x^2} = \sqrt{1.707}$

$\boxed{x = \pm 1.307}$

4.2 Day 2 Quotient Rule

#11-27, 29, 38

(11) $f'(x) = \frac{6(3x+10) - (6x+1)(3)}{(3x+10)^2} = \frac{18x+60-18x-3}{(3x+10)^2} = \boxed{\frac{57}{(3x+10)^2}}$

(12) $f'(x) = \frac{8(7x+3) - (8x-11)(7)}{(7x+3)^2} = \frac{56x+24-56x+77}{(7x+3)^2} = \boxed{\frac{101}{(7x+3)^2}}$

(13) $\frac{dy}{dx} = \frac{-3(4+t) - (5-3t)(1)}{(4+t)^2} = \frac{-12-3t-5+3t}{(4+t)^2} = \boxed{\frac{-17}{(4+t)^2}}$

(14) $\frac{dy}{dx} = \frac{(-7)(1-t) - (9-7t)(-1)}{(1-t)^2} = \frac{-7+7t+9-7t}{(1-t)^2} = \boxed{\frac{2}{(1-t)^2}}$

(15) $\frac{dy}{dx} = \frac{(2x+1)(x-1) - (x^2+x)(1)}{(x-1)^2} = \frac{2x^2-x-1-x^2-x}{(x-1)^2} = \boxed{\frac{x^2-2x-1}{(x-1)^2}}$

(16) $\frac{dy}{dx} = \frac{(2x-4)(x+3) - (x^2-4x)(1)}{(x+3)^2} = \frac{2x^2+2x-12-x^2+4x}{(x+3)^2} = \boxed{\frac{x^2+6x-12}{(x+3)^2}}$

(17) $f'(t) = \frac{(8t)(t^2+3) - (4t^2+11)(2t)}{(t^2+3)^2} = \frac{8t^3+24t-8t^3-22t}{(t^2+3)^2} = \boxed{\frac{2t}{(t^2+3)^2}}$

(18) $\frac{dy}{dx} = \frac{(-2x+8)(4x^2-5) - (-x^2+8x)(8x)}{(4x^2-5)^2} = \frac{-8x^3+10x+32x^2-40+8x^3-64x^2}{(4x^2-5)^2}$

$= \boxed{\frac{-32x^2+10x-40}{(4x^2-5)^2}}$

(19) $g'(x) = \dfrac{(2x-4)(x^2+3) - (x^2-4x+2)(2x)}{(x^2+3)^2}$

$= \dfrac{2x^3+6x-4x^2-12-2x^3+8x^2-4x}{(x^2+3)^2} = \boxed{\dfrac{4x^2+2x-12}{(x^2+3)^2}}$

(20) $k'(x) = \dfrac{(2x+7)(x^2-2) - (x^2+7x-2)(2x)}{(x^2-2)^2}$

$= \dfrac{2x^3-4x+7x^2-14-2x^3-14x^2+4x}{(x^2-2)^2} = \boxed{\dfrac{-7x^2-14}{(x^2-2)^2}}$

(21) $p'(t) = \dfrac{\left(\frac{1}{2}t^{-1/2}\right)(t-1) - t^{1/2}(1)}{(t-1)^2} = \dfrac{\frac{1}{2}t^{1/2} - \frac{1}{2}t^{-1/2} - t^{1/2}}{(t-1)^2}$

$= \dfrac{-\frac{1}{2}t^{1/2} - \frac{1}{2}t^{-1/2}}{(t-1)^2} = \dfrac{-\frac{1}{2}\left(t^{1/2} + t^{-1/2}\right)}{(t-1)^2}$ $\boxed{\dfrac{\sqrt{t} - \frac{1}{\sqrt{t}}}{-2(t-1)^2}}$

$\boxed{\dfrac{-\frac{\sqrt{t}}{2} - \frac{1}{2\sqrt{t}}}{(t-1)^2}}$ or Something

(22) $r'(t) = \dfrac{\left(\frac{1}{2}t^{-1/2}\right)(2t+3) - t^{1/2}(2)}{(2t+3)^2} = \dfrac{t^{1/2} + \frac{3}{2}t^{-1/2} - 2t^{1/2}}{(2t+3)^2}$

$= \boxed{\dfrac{\frac{3}{2\sqrt{t}} - \sqrt{t}}{(2t+3)^2}}$

(23) $\frac{dy}{dx} = \frac{5(x^{1/2}) - (5x+6)(\frac{1}{2}x^{-1/2})}{(\sqrt{x})^2} = \frac{5x^{1/2} - \frac{5}{2}x^{1/2} - 3x^{-1/2}}{x}$

$= \frac{\frac{5}{2}x^{1/2} - 3x^{-1/2}}{x} = \boxed{\frac{\frac{5\sqrt{x}}{2} - \frac{3}{\sqrt{x}}}{x}} \text{ or } \boxed{\frac{5}{2\sqrt{x}} - \frac{3}{\sqrt{x^3}}}$

(24) $h'(z) = \frac{2.2z^{1.2}(z^{3.2}+5) - z^{2.2}(3.2z^{2.2})}{(z^{3.2}+5)^2} = \frac{2.2z^{4.4} + 11z^{1.2} - 3.2z^{4.4}}{(z^{3.2}+5)^2}$

$= \boxed{\frac{11z^{1.2} - z^{4.4}}{(z^{3.2}+5)^2}}$

(25) $g'(y) = \frac{(1.4y^{1.4})(y^{2.5}+2) - (y^{1.4}+1)(2.5y^{1.5})}{(y^{2.5}+2)^2} = \frac{1.4y^{2.9} + 2.8y^{1.4} - 2.5y^{2.9} - 2.5y^{1.5}}{(y^{2.5}+2)^2}$

$= \boxed{\frac{-1.1y^{2.9} - 2.5y^{1.5} + 2.8y^{1.4}}{(y^{2.5}+2)^2}}$

(26) $f'(x) = \frac{[6x(2x-1) + (3x^2+1)(2)](5x+4) - (3x^2+1)(2x-1)(5)}{(5x+4)^2}$

$= \frac{(12x^2 - 6x + 6x^2 + 2)(5x+4) - (6x^3 - 3x^2 + 2x - 1)(5)}{(5x+4)^2}$

$= \frac{(18x^2 - 6x + 2)(5x+4) - 30x^3 + 15x^2 - 10x + 5}{(5x+4)^2}$

$= \frac{90x^3 + 72x^2 - 30x^2 - 24x + 10x + 8 - 30x^3 + 15x^2 - 10x + 5}{(5x+4)^2}$ ↘

26 continued

$$\boxed{F'(x) = \frac{60x^3 + 57x^2 - 24x + 13}{(5x+4)^2}}$$

(27) $$g'(x) = \frac{\left[4x(5x+2) + (2x^2+3)(5)\right](6x-7) - (2x^2+3)(5x+2)(6)}{(6x-7)^2}$$

$$= \frac{\left[20x^2 + 8x + 10x^2 + 15\right](6x-7) - (10x^3 + 4x^2 + 15x + 6)(6)}{(6x-7)^2}$$

$$= \frac{(30x^2 + 8x + 15)(6x-7) - 60x^3 - 24x^2 - 90x - 36}{(6x-7)^2}$$

$$= \frac{180x^3 + 48x^2 + 90x - 210x^2 - 56x - 105 - 60x^3 - 24x^2 - 90x - 36}{(6x-7)^2}$$

$$\boxed{g'(x) = \frac{120x^3 - 186x^2 - 56x - 141}{(6x-7)^2}}$$

(29) $$h'(x) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2} \qquad h'(3) = \frac{f'(3)\cdot g(3) - f(3)\cdot g'(3)}{(g(3))^2}$$

$$h'(3) = \frac{8.4 - 9.5}{4^2} = \frac{32 - 45}{16} = \boxed{\frac{-13}{16}}$$

(38) $F'(x) = \frac{1(x^2+4) - (x-2)(2x)}{(x^2+4)^2} = \frac{x^2+4-2x^2+4x}{(x^2+4)^2}$

$F'(x) = \frac{-x^2+4x+4}{(x^2+4)^2} = 0 \rightarrow (-x^2+4x+4=0)\cdot -1$

$x^2-4x-4=0$

$x = \frac{4 \pm \sqrt{16-4(-4)}}{2} = \frac{4 \pm \sqrt{32}}{2}$

$\boxed{x = 4.828}$ & $\boxed{x = -.828}$