Calculus 4.3 chain Rule

Day 1 # 1-13 odd, 16-21, 22-27

(1) $g(2)=19$

$f(19)=5(19^2)-2(19)$

$\boxed{f(g(2))=1767}$

(3) $f(2)=5(2^2)-2(2)$

$f(2)=16$

$g(16)=8(16)+3=\boxed{131=g(f(2))}$

(5) $g(k)=8k+3$

$f(g(k))=5(8k+3)^2-2(8k+3)=5(64k^2+48k+9)-16k-6$

$=320k^2+240k+45-16k-6$

$\boxed{f(g(k))=320k^2+224k+39}$

(7) $\boxed{f(g(x))=\frac{6x-1}{8}+7}$ $\boxed{g(f(x))=6\left(\frac{x}{8}+7\right)-1=\frac{3}{4}x+41}$

(9) $\boxed{f(g(x))=\frac{1}{x^2}}$ $\boxed{g(f(x))=\left(\frac{1}{x}\right)^2=\frac{1}{x^2}}$

(11) $f(g(x))=\sqrt{8x^2-6+2}=\boxed{\sqrt{8x^2-4}}$

$g(f(x))=8\left(\sqrt{x+2}\right)^2-6$

$=8(x+2)-6=\boxed{8x+10}$

(13) $\boxed{f(g(x))=\sqrt{\frac{-1}{x}+1}}$ $\boxed{g(f(x))=\frac{-1}{\sqrt{x+1}}}$

(16) $f(x)=x^{2/3}$ $g(x)=3x^2-7$ $y=f(g(x))$

(17) $f(x)=x^{3/5}$ $g(x)=5-x^2$ $y=f(g(x))$

(18) $f(x) = \sqrt{x} \quad g(x) = 9 - 4x \quad y = f(g(x))$

(19) $f(x) = -\sqrt{x} \quad g(x) = 13 + 7x \quad y = f(g(x))$

(20) $f(x) = x^2 + x + 5 \quad g(x) = x^{\frac{1}{2}} - 3 \quad y = f(g(x))$

(21) $f(x) = x^{1/3} - 2x^{2/3} + 7 \quad g(x) = x^2 + 5x \quad y = f(g(x))$

(22) $y = (2x^3 + 9x)^5$

$\frac{dy}{dx} = 5(2x^3+9x)^4 \cdot (6x^2+9) = \boxed{(30x^2+45)(2x^3+9x)^4}$

(23) $y = (8x^4 - 5x^2 + 1)^4$

$\frac{dy}{dx} = 4(8x^4 - 5x^2 + 1)^3 \cdot (32x^3 - 10x)$

$\boxed{\frac{dy}{dx} = (128x^3 - 40x)(8x^4 - 5x^2 + 1)^3}$

(24) $f(x) = -7(3x^4 + 2)^{-4}$

$f'(x) = 28(3x^4+2)^{-5}(12x^3) = \boxed{\dfrac{336x^3}{(3x^4+2)^5}}$

(25) $k(x) = -2(12x^2+5)^{-6}$

$k'(x) = 12(12x^2+5)^{-7} \cdot (24x) = \boxed{\dfrac{288x}{(12x^2+5)^7}}$

26) $S(t) = 12(2t^4+5)^{3/2}$

$S'(t) = 18(2t^4+5)^{1/2}\cdot(8t) = \boxed{144t\cdot\sqrt{2t^4+5}}$

27) $s(t) = 45(3t^3-8)^{3/2}$

$S'(t) = \frac{135}{2}(3t^3-8)^{1/2}\cdot(9t^2) = \boxed{\frac{1215}{2}t^2\sqrt{3t^3-8}}$

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28) $f(t) = 8(4t^2+7)^{1/2}$

$f'(t) = 4(4t^2+7)^{-1/2}\cdot(8t) = \boxed{\frac{32t}{\sqrt{4t^2+7}}}$

29) $g(t) = -3(7t^3-1)^{1/2}$

$g'(t) = \frac{-3}{2}(7t^3-1)^{-1/2}\cdot(21t^2) = \boxed{\frac{-63t^2}{2\sqrt{7t^3-1}}}$

30) $r(t) = 4t(2t^5+3)^4$ ← use Product Rule

$r'(t) = 4(2t^5+3)^4 + (4t)(4)(2t^5+3)^3(10t^4)$

$= 4(2t^5+3)^3[2t^5+3+40t^5]$

$\boxed{= 4(2t^5+3)^3[42t^5+3] \text{ or } = (2t^5+3)^3[168t^5+12]}$

31) $m(t) = -6t(5t^4-1)^4$

$m'(t) = -6(5t^4-1)^4 + (-6t)(4)(5t^4-1)^3(20t^3)$

$= -6(5t^4-1)^3[5t^4-1+80t^4] = \boxed{-6(5t^4-1)^3[85t^4-1] \text{ or } (-510t^4+6)(5t^4-1)^3}$

(32) $y = (x^3+2)(x^2-1)^4$

$\frac{dy}{dx} = 3x^2(x^2-1)^4 + (x^3+2)(4)(x^2-1)^3(2x)$

$= x(x^2-1)^3\left[3x(x^2-1) + 8x^3+16\right]$

$= x(x^2-1)^3\left[3x^3 - 3x + 8x^3 + 16\right]$

$\boxed{= x(x^2-1)^3\left[11x^3 - 3x + 16\right]}$

(33) $y = (3x^4+1)^4(x^3+4)$

$\frac{dy}{dx} = 4(3x^4+1)^3(12x^3)(x^3+4) + (3x^4+1)^4(3x^2)$

$= 3x^2(3x^4+1)^3\left[16x(x^3+4) + 3x^4+1\right]$

$= 3x^2(3x^4+1)^3\left[16x^4 + 64x + 3x^4 + 1\right]$

$\boxed{\frac{dy}{dx} = 3x^2(3x^4+1)^3\left[19x^4 + 64x + 1\right]}$

(34) $P(z) = z(6z+1)^{4/3}$

$P'(z) = (6z+1)^{4/3} + z\left(\frac{4}{3}\right)(6z+1)^{1/3}(6)$

$= (6z+1)^{1/3}\left[6z+1 + 8z\right]$

$= (6z+1)^{1/3}\left[14z+1\right] \qquad \boxed{= \sqrt[3]{6z+1}\,(14z+1)}$

(35) $g(y) = 4y^2(y^2+1)^{5/4}$

$g'(y) = 8y(y^2+1)^{5/4} + 4y^2\left(\frac{5}{4}\right)(y^2+1)^{1/4}(2y)$

$= 8y(y^2+1)^{5/4} + 10y^3(y^2+1)^{1/4}$

$= 2y(y^2+1)^{1/4}\left[4(y^2+1) + 5y^2\right]$

$\boxed{g'(y) = 2y(y^2+1)^{1/4}\left[9y^2 + 4\right]}$

(36) $y = \dfrac{1}{(3x^2-4)^5}$

$$\frac{dy}{dx} = \frac{0(3x^2-4)^5 - (1)(5)(3x^2-4)^4(6x)}{(3x^2-4)^{10}} = \frac{-30x(3x^2-4)^4}{(3x^2-4)^{10}}$$

$$\boxed{\frac{dy}{dx} = \frac{-30x}{(3x^2-4)^6}}$$

(37) $y = \dfrac{-5}{(2x^3+1)^2}$

$$\frac{dy}{dx} = \frac{0(2x^3+1)^2 - (-5)(2)(2x^3+1)(6x^2)}{(2x^3+1)^4} = \frac{60x^2(2x^3+1)}{(2x^3+1)^4}$$

$$\boxed{\frac{dy}{dx} = \frac{60x^2}{(2x^3+1)^3}}$$

Another way to do this problem is

$$y = -5(2x^3+1)^{-2}$$

$$\frac{dy}{dx} = 10(2x^3+1)^{-3}(6x^2) = \frac{60x^2}{(2x^3+1)^3}$$

(38) $p(t) = \dfrac{(2t+3)^3}{4t^2-1}$

$$p'(t) = \frac{3(2t+3)^2(2)(4t^2-1) - (2t+3)^3(8t)}{(4t^2-1)^2}$$

$$p'(t) = \frac{2(2t+3)^2[12t^2-3-8t^2-12t]}{(4t^2-1)^2}$$

$$\boxed{p'(t) = \frac{2(2t+3)^2[4t^2-12t-3]}{(4t^2-1)^2}}$$

39 $r(t) = \dfrac{(5t-6)^4}{3t^2+4}$

$$r'(t) = \frac{4(5t-6)^3(5)(3t^2+4) - (5t-6)^4(6t)}{(3t^2+4)^2}$$

$$= \frac{20(5t-6)^3(3t^2+4) - (6t)(5t-6)^4}{(3t^2+4)^2}$$

$$= \frac{2(5t-6)^3\left[10(3t^2+4) - 3t(5t-6)\right]}{(3t^2+4)^2}$$

$$= \frac{2(5t-6)^3\left[30t^2+40-15t^2+18t\right]}{(3t^2+4)^2}$$

$$\boxed{r'(t) = \frac{2(5t-6)^3\left[15t^2+18t+40\right]}{(3t^2+4)^2}}$$

40 $y = \dfrac{x^2+4x}{(3x^3+2)^4}$

$$\frac{dy}{dx} = \frac{(2x+4)(3x^3+2)^4 - (x^2+4x)(4)(3x^3+2)^3(9x^2)}{(3x^3+2)^8}$$

$$= \frac{(3x^3+2)^3\left[(2x+4)(3x^3+2) - 36x^2(x^2+4x)\right]}{(3x^3+2)^8}$$

$$= \frac{6x^4+12x^3+4x+8-36x^4-144x^3}{(3x^3+2)^5}$$

$$\boxed{\frac{dy}{dx} = \frac{-30x^4-132x^3+4x+8}{(3x^3+2)^5}}$$

(41) $y = \dfrac{3x^2 - x}{(2x-1)^5}$

$$\frac{dy}{dx} = \frac{(6x-1)(2x-1)^5 - (3x^2-x)(5)(2x-1)^4(2)}{(2x-1)^{10}}$$

$$= \frac{(2x-1)^4\left[(6x-1)(2x-1) - (10)(3x^2-x)\right]}{(2x-1)^{10}}$$

$$= \frac{12x^2 - 2x - 6x + 1 - 30x^2 + 10x}{(2x-1)^6} = \boxed{\frac{-18x^2 + 2x + 1}{(2x-1)^6}}$$