Calculus - Chain Rule - Story Problems

(53) Demand in terms of cost means a composite $D(p(c))$

$$\boxed{D(c) = \frac{-(2c-10)^2}{100} + 500}$$

(54) $R'(x) = 16(x^2+x)^{-1/3}(2x+1) = \dfrac{16(2x+1)}{\sqrt[3]{x^2+x}}$

a) $R'(100) = \dfrac{(16)(2(100)+1)}{\sqrt[3]{100^2+100}} = \$148.78$ per set

b) $R'(200) = \$187.29$ per set

c) $R'(300) = \$214.34$ per set

d) Average Revenue $= \dfrac{\text{Revenue}}{\#\text{ of sets}} = \boxed{\dfrac{24(x^2+x)^{2/3}}{x}} = \overline{R(x)}$

e) $\overline{R(x)}' = \dfrac{16(x^2+x)^{-1/3}(2x+1)x - 24(x^2+x)^{2/3}}{x^2}$

$$\frac{8(x^2+x)^{-1/3}\left[4x^2+2x-3x^2-3x\right]}{x^2}$$

$$\boxed{\overline{R(x)}' = \frac{8(x^2+x)^{-1/3}\left[x^2-x\right]}{x^2}}$$

55 $A' = 2737500\left(1+\frac{r}{36500}\right)^{1824}\left(\frac{1}{36500}\right)$

$A'(r) = 75\left(1+\frac{r}{36500}\right)^{1824}$ Plug in %

a) $A'(6) = 75\left(1+\frac{6}{36500}\right)^{1824}$

$= \$101.22$

b) $A'(8) = \$111.86$

c) $A'(9) = \$117.59$

56 $D(p) = 150 - \frac{30p}{(p^2+1)^{1/2}}$

$D'(p) = 0 - \frac{30(p^2+1)^{1/2} - (30p)\left(\frac{1}{2}\right)(p^2+1)^{-1/2}(2p)}{\left[(p^2+1)^{1/2}\right]^2}$

$D'(p) = -\frac{30(p^2+1)^{-1/2}\left[(p^2+1) - p^2\right]}{(p^2+1)^1} = \boxed{\frac{-30}{(p^2+1)^{3/2}}}$

60 a) $A(r(t)) = \pi(t^2)^2 = \pi \cdot t^4$

$A(r(t))$ is the area of the spill @ a time t

b) $D_t A[r(t)] = A'(t) = 4\pi t^3$

$A'(100) = 4\pi(100)^3 = 4{,}000{,}000\pi$

$\approx 12{,}566{,}370.61$ Ft^2/min @ 100 min

(62) $N'(t) = 2(5t+9)^{1/2} + \cancel{(2t)}(\tfrac{1}{2})\cancel{(5t+9)}^{-1/2}(5) \quad + 0$

$N'(t) = (5t+9)^{-1/2}\left[10t+18 + 5t\right] = (5t+9)^{-1/2}\left[15t+18\right]$

$N'(t) = \dfrac{15t+18}{\sqrt{5t+9}}$

a) $N'(0) = \boxed{6}$ b) $N'\left(\frac{7}{5}\right) = \dfrac{21+18}{\sqrt{7+9}} = \boxed{\dfrac{39}{4}}$ c) $N'(8) = \boxed{\dfrac{138}{7}}$

(65) a) Life expectancy is when $r=0$

$0 = 6 - \frac{3}{17}t$

$\frac{3}{17}t = 6$

$\boxed{t \approx 34 \text{ minutes}}$

b) $V(r(t)) = \frac{4}{3}\pi\left(6 - \frac{3}{17}t\right)^3$

$\dfrac{dV}{dt} = 4\pi\left(6-\frac{3}{17}t\right)^2\left(\frac{-3}{17}\right)$

$\dfrac{dV}{dt} = \dfrac{-12}{17}\pi\left(6-\frac{3}{17}t\right)^2$

$\dfrac{dV}{dt}$ @ $t=17 \rightarrow \dfrac{-108\pi}{17} \dfrac{mm^3}{min}$

≈ -19.958

the volume is decreasing by 19.958 mm^3 per min

$S(r(t)) = 4\pi\left(6 - \frac{3}{17}t\right)^2$

$\dfrac{dS}{dt} = 8\pi\left(6-\frac{3}{17}t\right)\left(\frac{-3}{17}\right)$

$\dfrac{dS}{dt} = \dfrac{-24\pi}{17}\left(6-\frac{3}{17}t\right)$

$\dfrac{dS}{dt}$ @ $t=17 \rightarrow \dfrac{-72\pi}{17} \approx -13.31$

Surface area is decreasing by 13.31 mm^2 per min