

CALCULUS 4.4 Der. of EXP.

Notes

$$\begin{aligned}
 * F(x) &= e^x & F'(x) &= e^x \\
 F(x) &= a^x & F'(x) &= (\ln a) a^x \\
 F(x) &= a^{g(x)} & F'(x) &= (\ln a) \cdot a^{g(x)} \cdot g'(x) \\
 F(x) &= e^{g(x)} & F'(x) &= e^{g(x)} \cdot g'(x)
 \end{aligned}$$

H.W.

$$\begin{aligned}
 ① \quad y &= e^{4x} & \frac{dy}{dx} &= 4e^{4x} \\
 ② \quad y &= e^{-2x} & \frac{dy}{dx} &= -2e^{-2x} \\
 ③ \quad y &= -8e^{3x} & \frac{dy}{dx} &= -24e^{3x} \\
 ④ \quad y &= 1.2e^{5x} & \frac{dy}{dx} &= 6e^{5x}
 \end{aligned}$$

$$\begin{aligned}
 ⑤ \quad y &= -16e^{2x+1} & \frac{dy}{dx} &= -32e^{2x+1} \\
 ⑥ \quad y &= -4e^{-3x} & \frac{dy}{dx} &= 1.2e^{-3x} \\
 ⑦ \quad y &= e^{x^2} & \frac{dy}{dx} &= 2x \cdot e^{x^2} \\
 ⑧ \quad y &= e^{-x^2} & \frac{dy}{dx} &= -2x \cdot e^{-x^2}
 \end{aligned}$$

$$\begin{aligned}
 ⑨ \quad y &= 3e^{2x^2} & \frac{dy}{dx} &= 12xe^{2x^2} \\
 ⑩ \quad y &= -5e^{4x^3} & \frac{dy}{dx} &= -60x^2e^{4x^3} \\
 ⑪ \quad y &= 4e^{2x^2-4} & \frac{dy}{dx} &= 16xe^{2x^2-4}
 \end{aligned}$$

$$\begin{aligned}
 ⑫ \quad y &= -3e^{3x^2+5} & \frac{dy}{dx} &= -18xe^{3x^2+5} \\
 ⑬ \quad y &= xe^x & \frac{dy}{dx} &= e^x + xe^x \\
 & & &= e^x(1+x) \\
 ⑭ \quad y &= x^2e^{-2x} & \frac{dy}{dx} &= 2xe^{-2x} + -2x^2e^{-2x} \\
 & & &= 2xe^{-2x}(1-x)
 \end{aligned}$$

$$⑮ \quad y = (x+3)^2 e^{4x} \\
 \frac{dy}{dx} = 2(x+3)(1)e^{4x} + (x+3)^2(4)e^{4x} = 2(x+3)e^{4x}(x+7)$$

$$⑯ \quad y = (3x^3 - 4x)e^{-5x} \\
 \frac{dy}{dx} = (9x^2 - 4)e^{-5x} + (3x^3 - 4x)(-5)e^{-5x} = e^{-5x}(9x^2 - 4 - 15x^3 + 20x) \\
 = e^{-5x}(-15x^3 + 9x^2 + 20x - 4)$$

17) $y = \frac{x^2}{e^x}$

$\frac{dy}{dx} = \frac{2xe^x - x^2e^x}{e^{2x}} = \frac{xe^x(2-x)}{e^{2x}} = \frac{x(2-x)}{e^x}$

18) $y = \frac{e^x}{2x+1}$ $\frac{dy}{dx} = \frac{e^x(2x+1) - 2e^x}{(2x+1)^2} = \frac{e^x(2x+1-2)}{(2x+1)^2} = \frac{e^x(2x-1)}{(2x+1)^2}$

19) $y = \frac{e^x + e^{-x}}{x}$ $\frac{dy}{dx} = \frac{(e^x - e^{-x})x - (e^x + e^{-x})}{x^2}$ $1^x - x e^k - e^x - e^{-x}$

20) $y = \frac{e^x - e^{-x}}{x}$ $\frac{dy}{dx} = \frac{(e^x + e^{-x})x - (e^x - e^{-x})}{x^2}$

21) $P = \frac{10000}{9 + 4e^{-.2t}}$ $\frac{dP}{dt} = \frac{0 - 10000(-.2e^{-.2t})}{(9 + 4e^{-.2t})^2}$
 $= \frac{20000e^{-.2t}}{(9 + 4e^{-.2t})^2}$

22) $P = \frac{500}{12 + 5e^{-.5t}}$ $\frac{dP}{dt} = \frac{0 - 500(-2.5e^{-.5t})}{(12 + 5e^{-.5t})^2}$
 $= \frac{1250e^{-.5t}}{(12 + 5e^{-.5t})^2}$

23) $F(z) = (2z + e^{-z^2})^2$ $F'(z) = 2(2z + e^{-z^2})(2 - 2ze^{-z^2})$

24) $y = 7^{3x+1}$ $\frac{dy}{dx} = (\ln 7) 7^{3x+1} \cdot 3 = 3 \cdot 7^{3x+1} (\ln 7)$

25) $y = 4^{-5x+2}$ $\frac{dy}{dx} = (\ln 4) 4^{-5x+2} (-5) = -5(4^{-5x+2})(\ln 4)$

26) $y = 3 \cdot 4^{x^2+2}$ $= 3(\ln 4)(4^{x^2+2}) 2x = 6x(\ln 4) 4^{x^2+2}$

$$(27) \quad y = -10^{3x^2-4} \quad \frac{dy}{dx} = (\ln 10)(6x) 10^{3x^2-4}$$

$$(28) \quad s = 2 \cdot 3^{\sqrt{t}} \quad \frac{ds}{dt} = 2(\ln 3) 3^{\sqrt{t}} \left(\frac{1}{2\sqrt{t}}\right) = \frac{(\ln 3) \cdot 3^{\sqrt{t}}}{\sqrt{t}}$$

$$(29) \quad s = 5 \cdot 2^{\sqrt{t-2}} = 5 \cdot 2^{(t-2)^{1/2}} = 5(\ln 2) \cdot 2^{(t-2)^{1/2}} \cdot \frac{1}{2}(t-2)^{-1/2} \\ = \frac{5(\ln 2) 2^{\sqrt{t-2}}}{2\sqrt{t-2}}$$

$$(30) \quad y = \frac{te^t + 2}{e^{2t} + 1} \quad \frac{dy}{dt} = \frac{(e^t + te^t)(e^{2t} + 1) - (te^t + 2)(2e^{2t})}{(e^{2t} + 1)^2} \\ = \frac{e^{3t}(1-t) + e^t(1+t) - 4e^{2t}}{(e^{2t} + 1)^2}$$

$$(31) \quad y = \frac{te^{2t}}{t + e^{3t}} \quad \frac{dy}{dx} = \frac{(2te^{2t} + t^2e^{2t})(t + e^{3t}) - (te^{2t})(1 + 3e^{3t})}{(t + e^{3t})^2} \\ = \frac{(2te^{2t})(1+t)(t + e^{3t}) - (te^{2t})(1 + 3e^{3t})}{(t + e^{3t})^2}$$

$$t + t^2 + e^{3t} + te^{3t} \\ 2t + 2t^2 + 2e^{3t} + 2te^{3t} - t - 3e^{3t} \\ te^{2t} (2t + 2t^2 - e^{3t} + 2te^{3t} - 1) \\ 2t^2e^{2t} + 2te^{2t} - te^{5t} + 2t^2e^{5t} - te^{2t}$$

$$(31) \frac{dy}{dx} = \frac{(2te^{2t} + 2t^2e^{3t})(t+e^{3t}) - (t^2e^{2t})(1+3e^{3t})}{(t+e^{3t})^2}$$

$$= \frac{2t^2e^{2t} + 2te^{2t}e^{3t} + 2te^{5t} + 2t^2e^{3t}e^{3t} - t^2e^{2t} - 3t^2e^{5t}}{(t+e^{3t})^2}$$

$$= \frac{te^{2t} + 2te^{3t} + 2te^{5t} - t^2e^{5t}}{(t+e^{3t})^2}$$

$$= \frac{te^{2t}(1+2t) + te^{5t}(2-t)}{(t+e^{3t})^2}$$

$$(32) f(x) = e^{x\sqrt{3x+2}} \quad f'(x) = \left(\sqrt{3x+2} + x \left(\frac{3}{2} \right) \frac{1}{\sqrt{3x+2}} \right) e^{x\sqrt{3x+2}}$$

$$\frac{(3x+2)^2}{2(3x+2)^{3/2}}$$

$$= \left(\frac{3x+2 + \frac{3}{2}x}{\sqrt{3x+2}} \right) e^{x\sqrt{3x+2}}$$

$$= \left(\frac{\frac{9}{2}x + 2}{\sqrt{3x+2}} \right) e^{x\sqrt{3x+2}}$$

$$(33) f(x) = e^{\frac{x^2}{x^3+2}}$$

$$f'(x) = \frac{2x(x^3+2) - x^2(3x^2)}{(x^3+2)^2} e^{\frac{x^2}{x^3+2}}$$

$$= \frac{2x^4 + 4x - 3x^4}{(x^3+2)^2} e^{\frac{x^2}{x^3+2}}$$

$$= \left(\frac{-x^4 + 4x}{(x^3+2)^2} \right) e^{\frac{x^2}{x^3+2}}$$