# Chapter 4 Review

Concept check

① F ② T ③ F ④ T ⑤ F ⑥ T ⑦ F ⑧ T ⑨ F ⑩ F

① $\boxed{\frac{dy}{dx} = 15x^2 - 14x - 9}$

② $\boxed{\frac{dy}{dx} = 21x^2 - 8x - 5}$

③ $\boxed{\frac{dy}{dx} = 24x^{5/3}}$

④ $\frac{dy}{dx} = 12x^{-4} = \boxed{\frac{12}{x^4}}$

⑤ $F'(x) = -12x^{-5} + 6\frac{1}{2\sqrt{x}} = \boxed{\frac{-12}{x^5} + \frac{3}{\sqrt{x}}}$

⑥ $F'(x) = -19x^{-2} - \frac{8}{2\sqrt{x}} = \boxed{\frac{-19}{x^2} - \frac{4}{\sqrt{x}}}$

⑦ $k'(x) = \frac{3(4x+7) - (3x)(4)}{(4x+7)^2} = \frac{12x + 21 - 12x}{(4x+7)^2}$

$\boxed{k'(x) = \frac{21}{(4x+7)^2}}$

⑧ $r'(x) = \frac{-8(2x+1) - (-8x)(2)}{(2x+1)^2}$

$= \frac{-16x - 8 + 16x}{(2x+1)^2} = \boxed{\frac{-8}{(2x+1)^2}}$

⑨ $\frac{dy}{dx} = \frac{(2x-1)(x-1) - (x^2 - x + 1)(1)}{(x-1)^2}$

$= \frac{\cancel{2x^2} - x - 2x + 1 - \cancel{x^2} + x - 1}{(x-1)^2}$

$\boxed{\frac{dy}{dx} = \frac{x^2 - 2x}{(x-1)^2}}$

⑩ $\frac{dy}{dx} = \frac{(6x^2 - 10x)(x+2) - (2x^3 - 5x^2)(1)}{(x+2)^2}$

$= \frac{\cancel{6x^3} + 12x^2 - 10x^2 - 20x - \cancel{2x^3} + 5x^2}{(x+2)^2}$

$\boxed{\frac{dy}{dx} = \frac{4x^3 + 7x^2 - 20x}{(x+2)^2}}$

⑪ $f'(x) = 4(3x^2 - 2)^3(6x)$

$\boxed{F'(x) = 24x(3x^2 - 2)^3}$

⑫ $k'(x) = 6(5x^3 - 1)^5(15x^2)$

$\boxed{k'(x) = 95x^2(5x^3 - 1)^5}$

⑬ $y = (2t^7 - 5)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(2t^7 - 5)^{-1/2}(14t^6)$

$\boxed{\frac{dy}{dx} = \frac{7t^6}{\sqrt{2t^7 - 5}}}$

⑭ $y = -3(8t^4 - 1)^{1/2}$

$\frac{dy}{dt} = \frac{-3}{2}(8t^4 - 1)^{-1/2}(32t^3)$

$\boxed{\frac{dy}{dt} = \frac{-48t^3}{\sqrt{8t^4 - 1}}}$

(15) $\dfrac{dy}{dx} = 3(2x+1)^{3} + (3x)(3)\cancel{(2x+1)^{2}}(2)$

$= 3(2x+1)^2\left[2x+1 + 6x\right]$

$\boxed{\dfrac{dy}{dx} = 3(2x+1)^2\left[8x+1\right]}$

(16) $\dfrac{dy}{dx} = 8x(3x-2)^{5} + 4x^{2}(5)\cancel{(3x-2)^{4}}(3)$

$= 4x(3x-2)^4\left[6x-4 + 15x\right]$

$\boxed{\dfrac{dy}{dx} = 4x(3x-2)^4\left[21x-4\right]}$

(17) $r'(t) = \dfrac{(10t-7)(3t+1)^3 - (5t^2-7t)(3)\cancel{(3t+1)^2}(3)}{(3t+1)^6}$

$= \dfrac{\cancel{(3t+1)^2}\left[(10t-7)(3t+1) - 45t^2 + 63t\right]}{(3t+1)^{\cancel{6}\,4}}$

$= \dfrac{30t^2 + 10t - 21t - 7 - 45t^2 + 63t}{(3t+1)^4} = \boxed{\dfrac{-15t^2 + 52t - 7}{(3t+1)^4}}$

(18) $s'(t) = \dfrac{(3t^2-2)(4t-3)^4 - (t^3-2t)(4)\cancel{(4t-3)^3}(4)}{(4t-3)^8}$

$= \dfrac{(4t-3)^3\left[(3t^2-2)(4t-3) - 16t^3 + 32t\right]}{(4t-3)^8}$

$= \dfrac{12t^3 - 9t^2 - 8t + 6 - 16t^3 + 32t}{(4t-3)^5} = \boxed{\dfrac{-4t^3 - 9t^2 + 24t + 6}{(4t-3)^5}}$

(19) $p'(t) = \cancel{(2t)}(t^2+1)^{5/2} + (t^2)\frac{5}{2}\cancel{(t^2+1)^{3/2}}\cancel{(2t)}$

$= (2t)(t^2+1)^{3/2}\left[t^2+1 + \frac{5}{2}t^2\right]$

$\boxed{p'(t) = (2t)(t^2+1)^{3/2}\left[\frac{7}{2}t^2 + 1\right]}$

could be written $\sqrt{(t^2+1)^3}$

(20) $g'(t) = (3t^2)(t^4+5)^{7/2} + (t^3)\left(\frac{7}{2}\right)\cancel{(t^4+5)^{5/2}}(4t^3)$

$= t^2(t^4+5)^{5/2}\left[3t^4+15 + 14t^4\right]$

$\boxed{g'(t) = t^2(t^4+5)^{5/2}\left[17t^4 + 15\right]}$

(21) $\frac{dy}{dx} = -6e^{2x}(2)$

$\boxed{\frac{dy}{dx} = -12e^{2x}}$

(22) $\frac{dy}{dx} = 8(e^{.5x})(.5)$

$\boxed{\frac{dy}{dx} = 4e^{.5x}}$

(23) $\frac{dy}{dx} = (e^{-2x^3})(-6x^2)$

$\boxed{\frac{dy}{dx} = -6x^2 \cdot e^{-2x^3}}$

(24) $\frac{dy}{dx} = -4(e^{x^2})(2x)$

$\boxed{\frac{dy}{dx} = -8x(e^{x^2})}$

(25) $\frac{dy}{dx} = 5e^{2x} + (5x)(e^{2x})(2)$

$\boxed{\frac{dy}{dx} = 5e^{2x}\left[1 + 2x\right]}$

(26) $\frac{dy}{dx} = (-14x)(e^{-3x}) + (-7x^2)(e^{-3x})(-3)$

$\boxed{\frac{dy}{dx} = -7xe^{-3x}\left[2 - 3x\right]}$

(27) $\boxed{\frac{dy}{dx} = \frac{2x}{2+x^2}}$

(28) $\boxed{\frac{dy}{dx} = \frac{5}{5x+3}}$

(29) $\frac{dy}{dx} = \frac{\left(\frac{3}{3x}\right)(x-3) - [\ln(3x)](1)}{(x-3)^2} = \frac{\frac{1}{x}(x-3) - \ln(3x)}{(x-3)^2} \cdot \frac{x}{x}$

$\boxed{\frac{dy}{dx} = \frac{x-3-x\ln(3x)}{x(x-3)^2}}$

(30) $\frac{dy}{dx} = \frac{\left(\frac{2}{2x-1}\right)(x+3) - [\ln(2x-1)](1)}{(x+3)^2} \cdot \frac{2x-1}{2x-1}$

$\boxed{\frac{dy}{dx} = \frac{2x+6 - (2x-1)[\ln(2x-1)]}{(2x-1)(x+3)^2}}$

(31) $$\frac{dy}{dx} = \frac{\left((1)e^x + xe^x\right)\left(\ln(x^2-1)\right) - (xe^x)\left(\frac{2x}{x^2-1}\right)}{\left[\ln(x^2-1)\right]^2}$$

$$= \frac{e^x(1+x)\left(\ln(x^2-1)\right) - x(e^x)\left(\frac{2x}{x^2-1}\right)}{\left[\ln(x^2-1)\right]^2} = \frac{e^x\left[(1+x)(\ln(x^2-1)) - \frac{2x^2}{x^2-1}\right]}{\left[\ln(x^2-1)\right]^2} \cdot \frac{x^2-1}{x^2-1}$$

$$\boxed{\frac{dy}{dx} = \frac{e^x\left[(x^2-1)(1+x)(\ln(x^2-1)) - 2x^2\right]}{(x^2-1)\left[\ln(x^2-1)\right]^2}}$$

(32) $$\frac{dy}{dx} = \frac{\left[(2x)(e^{2x}) + (x^2+1)(2)(e^{2x})\right](\ln x) - (x^2+1)(e^{2x})\left(\frac{1}{x}\right)}{\left[\ln x\right]^2}$$

$$= \frac{2e^{2x}\left[x + x^2+1\right](\ln x) - (x^2+1)(e^{2x})\left(\frac{1}{x}\right)}{\left[\ln x\right]^2} \cdot \frac{x}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{2xe^{2x}\left[x^2+x+1\right](\ln x) - (x^2+1)(e^{2x})}{x\left[\ln x\right]^2}}$$

(33) $$\boxed{s' = 2(t^2+e^t)(2t+e^t)}$$

(34) $$q' = 4\left(e^{2p+1}-2\right)^3\left((e^{2p+1})(2)\right)$$

$$\boxed{q' = 8(e^{2p+1})\left(e^{2p+1}-2\right)^3}$$

(35) $$\frac{dy}{dx} = 3(\ln 10)(10^{-x^2})(-2x)$$

$$\boxed{\frac{dy}{dx} = -6x(\ln 10)(10^{-x^2})}$$

(36) $$\frac{dy}{dx} = 10(\ln 2)(2^{\sqrt{x}})\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{10(\ln 2)(2^{\sqrt{x}})}{2\sqrt{x}} = \boxed{\frac{5(\ln 2)(2^{\sqrt{x}})}{\sqrt{x}}}$$

(37) $\boxed{g'(z) = \dfrac{3z^2+1}{(\ln 2)(z^3+z+1)}}$

(38) $\boxed{h'(z) = \dfrac{e^z}{(\ln 10)(1+e^z)}}$

(39) $F'(x) = (2e^{2x})\left[\ln(xe^x+1)\right] + (e^{2x})\left[\dfrac{(e^x+xe^x)}{xe^x+1}\right]$

$$\boxed{F'(x) = e^{2x}\left[2\ln(xe^x+1) + \frac{e^x+xe^x}{xe^x+1}\right]}$$

(40) $$F'(x) = \frac{e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)\left(\ln(\sqrt{x}+1)\right) - e^{\sqrt{x}}\left(\frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}+1}\right)}{\left[\ln(\sqrt{x}+1)\right]^2}$$

$$= \frac{\frac{e^{\sqrt{x}}}{2\sqrt{x}}\left[\ln(\sqrt{x}+1) - \frac{1}{\sqrt{x}+1}\right]}{\left[\ln(\sqrt{x}+1)\right]^2} = \boxed{\frac{e^{\sqrt{x}}\left[\ln(\sqrt{x}+1) - \frac{1}{\sqrt{x}+1}\right]}{\left[\ln(\sqrt{x}+1)\right]^2}}$$

(41) a) $D_x\left[F(g(x))\right] = F'(g(x)) \cdot g'(x)$

$D_2\left[F(g(x))\right] = F'(g(2)) \cdot g'(2) = F'(1) \cdot \frac{3}{10} = (-5)\left(\frac{3}{10}\right) = \boxed{\frac{-3}{2}}$

b) $D_x\left[F(g(x))\right] = F'(g(x)) \cdot g'(x)$

$D_3\left[F(g(x))\right] = F'(g(3)) \cdot g'(3) = F'(2) \cdot \frac{4}{11} = -6 \cdot \frac{4}{11} = \boxed{\frac{-24}{11}}$

(42) a) $D_2\left[g(F(x))\right] = g'(F(2)) \cdot F'(2) = g'(4) \cdot -6 = \frac{6}{13} \cdot -6 = \boxed{\frac{-36}{13}}$

b) $D_3\left[g(F(x))\right] = g'(F(3)) \cdot F'(3) = g'(2) \cdot (-7) = \frac{3}{10} \cdot -7 = \boxed{\frac{-21}{10}}$

44 → 53 I am not going to ask a question like this on the test, but here is what you need to do for question # 44

Step ① Slope of tangent line $\frac{dy}{dx} = 2x-6$ @ $x=2 \Rightarrow -2 = m$

Step ② Point of tangency — plug $x=2$ in function $y = 4-12 = -8$ $(2,-8)$

Step ③ Point-slope form of equation of line

$$y+8 = -2(x-2)$$
$$y = -2x+4-8$$
$$\boxed{y = -2x-4}$$

For questions 57–62 the average cost function is $\frac{C(x)}{x}$. The marginal average cost function is the derivative of $\frac{C(x)}{x}$

(57) $D_x\left[\frac{\sqrt{x+1}}{x}\right] = \frac{\frac{1}{2}(x+1)^{-1/2}(x) - (x+1)^{1/2}}{x^2} = \frac{(x+1)^{-1/2}\left[\frac{x}{2} - (x+1)\right]}{x^2} = \boxed{\frac{-1-\frac{x}{2}}{x^2\sqrt{x+1}}}$

(58) $D_x\left[\frac{\sqrt{3x+2}}{x}\right] = \frac{\frac{1}{2}(3x+2)^{-1/2}(3)x - (3x+2)^{1/2}}{x^2} = \frac{(3x+2)^{-1/2}\left[\frac{3x}{2} - (3x+2)\right]}{x^2} = \boxed{\frac{-\frac{3}{2}x-2}{x^2\sqrt{3x+2}}}$

(59) $D_x\left[\frac{(x^2+3)^3}{x}\right] = \frac{3(x^2+3)^2(2x)(x) - (x^2+3)^3(1)}{x^2} = \frac{(x^2+3)^2\left[6x^2-x^2-3\right]}{x^2} = \boxed{\frac{(x^2+3)^2\left[5x^2-3\right]}{x^2}}$

(60) $D_x\left[\frac{(4x+3)^4}{x}\right] = \frac{4(4x+3)^3(4)(x) - (4x+3)^4(1)}{x^2} = \frac{(4x+3)^3\left[16x-4x-3\right]}{x^2} = \frac{(4x+3)^3\left[12x-3\right]}{x^2}$

(61) $D_x\left[\frac{10-e^{-x}}{x}\right] = \frac{-e^{-x}(-1)x - (10-e^{-x})(1)}{x^2} = \boxed{\frac{xe^{-x} - 10 + e^{-x}}{x^2}}$

(62) $D_x\left[\frac{\ln(x+5)}{x}\right] = \frac{\frac{1}{x+5}\cdot x - \ln(x+5)}{x^2} = \boxed{\frac{\frac{x}{x+5} - \ln(x+5)}{x^2}}$

(63) $S'(x) = 60\left(\frac{1}{2\sqrt{x}}\right) + 12 = \frac{30}{\sqrt{x}} + 12$

a) $S'(9) = 22$

Sales increase by \$22 million Per \$1000 spent on research.

b) $S'(16) = 19.5$

Sales increase by \$19.5 million per \$1000 spent on research

c) $S'(25) = 18$

Sales increase by \$18 millions per \$1000 spent on research.

d) Sales derease.

(64) $P'(x) = \frac{(2x)(2x+1) - x^2(2)}{(2x+1)^2} = \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$

a) $P'(4) = \frac{40}{729}$

\$5.48 profit/unit

b) $P'(12) = .0199$

\$1.99 profit/unit

c) $P'(20) = .0122$

\$1.22 profit/unit

d) Profit decreases

e) $D_x\left[\frac{x^2}{x(2x+1)}\right] = D_x\left[\frac{x}{2x+1}\right] = \frac{2x+1-(x)(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2}$ @ $x=4$

$\frac{1}{81} \approx \$1.23$ average profit per unit

(65) $T'(x) = \frac{(60)(4x+5) - (1000+60x)(4)}{(4x+5)^2} = \frac{240x + 300 - 4000 - 240x}{(4x+5)^2} = \frac{-3700}{(4x+5)^2}$

a) $T'(9) = \frac{-3700}{1681} = -2.20$

The cost decreases by \$2,200 Per \$100 spent on training.

b) $T'(19) = -.56394$

The cost decreases by \$563.94 Per \$100 spent on training

c) Always decreasing.

(66) $\frac{dA}{dr} = 48000\left(1 + \frac{r}{400}\right)^{47}\left(\frac{1}{400}\right) = 120\left(1 + \frac{r}{400}\right)^{47}$

$\left.\frac{dA}{dr}\right|_{r=5} = 120\left(1 + \frac{5}{400}\right)^{47}$ $\boxed{\$215.15 = \text{change in value of account per year}}$

(67) $\frac{dA}{dr} = 1000\left(e^{12r/100}\right)\left(\frac{12}{100}\right) = 120e^{12r/100}$

$\left.\frac{dA}{dr}\right|_{r=5} = 120e^{60/100} = \boxed{\$218.65 = \text{change in account per year}}$

(68) $\frac{dT}{dr} = \frac{0(\ln(1+\frac{r}{100})) - (\ln 2)\left(\frac{\frac{1}{100}}{1+r/100}\right)}{\left[\ln(1+\frac{r}{100})\right]^2} = \frac{(-\ln 2)\left(\frac{1}{100+r}\right)}{\left[\ln(1+\frac{r}{100})\right]^2}$

$\frac{dT}{dr} = \frac{(-\ln 2)\left(\frac{1}{105}\right)}{\left[\ln(1+\frac{5}{100})\right]^2} = \boxed{-2.773 \Rightarrow \text{decreasing Doubling time per } r\%}$

(69) $F'(t) = 6.0828t^3 - 57.498t^2 + 125.82t + 6.0726$

$F'(7) \approx \boxed{155.811}$

(72) & (73) Don't worry about.