

Calculus 5.3

#1-25 (thirds)

① $F'(x) = 15x^2 - 14x + 4$

$F''(x) = 30x - 14$ $F''(0) = -14$ $F''(2) = 46$

④ $F'(x) = -4x^3 + 21x^2 - x$

$F''(x) = -12x^2 + 42x - 1$ $F''(0) = -1$ $F''(2) = 35$

⑦ $F'(x) = \frac{2x(1+x) - (x^2)(1)}{(1+x)^2} = \frac{2x + 2x^2 - x^2}{(1+x)^2} = \frac{2x + x^2}{(1+x)^2}$

$F''(x) = \frac{(2+2x)(1+x)^2 - (2x+x^2)(2)(1+x)(1)}{(1+x)^4}$

$F''(x) = \frac{(1+x) \overset{2+2x+2x+x^2}{(2+2x)(1+x) - 4x - 2x^2}}{(1+x)^4} = \frac{2}{(1+x)^3}$

$F''(0) = 2$ $F''(2) = \frac{2}{27}$

⑩ $F'(x) = \frac{1}{2}(2x^2+9)^{-1/2}(4x) = 2x(2x^2+9)^{-1/2}$

$F''(x) = 2(2x^2+9)^{-1/2} + (2x)(-\frac{1}{2})(2x^2+9)^{-3/2}(4x)$

$= 2(2x^2+9)^{-1/2} - 4x^2(2x^2+9)^{-3/2} = 2(2x^2+9)^{-3/2} [2x^2+9 - 2x^2]$

$F''(x) = \frac{18}{\sqrt{(2x^2+9)^3}}$

$F''(0) = \frac{18}{27} = \frac{2}{3}$

$F''(2) = \frac{18}{\sqrt{17^3}}$

$$\textcircled{13} F(x) = 5e^{-x^2}(-2x) = -10xe^{-x^2}$$

$$F'(x) = -10e^{-x^2} + (-10x)(e^{-x^2})(-2x) = -10e^{-x^2} + 20x^2e^{-x^2}$$

$$F''(x) = 10e^{-x^2}[-1 + 2x^2]$$

$$F''(0) = (10)(1)[-1] = -10$$

$$F''(2) = 10e^{-4}[-7] = \frac{70}{e^4}$$

$$\textcircled{16} F(x) = \ln x + x^{-1}$$

$$F'(x) = \frac{1}{x} - x^{-2}$$

$$F''(x) = -x^{-2} + 2x^{-3}$$

$$F''(x) = \frac{-1}{x^2} + \frac{2}{x^3}$$

$$F''(0) = \text{DNE}$$

$$F''(2) = \frac{-1}{4} + \frac{2}{8} = 0$$

$$\textcircled{19} F'(x) = 25x^4 - 12x^3 + 6x^2 + 14x$$

$$F''(x) = 100x^3 - 36x^2 + 12x + 14$$

$$F'''(x) = 300x^2 - 72x + 12$$

$$F^{(4)}(x) = 600x - 72$$

$$\textcircled{22} F'(x) = \frac{1(x) - (x+1)(1)}{x^2} = \frac{x - x - 1}{x^2} = \frac{-1}{x^2} = -x^{-2}$$

$$F''(x) = 2x^{-3}$$

$$F'''(x) = -6x^{-4} = \frac{-6}{x^4}$$

$$F^{(4)}(x) = 24x^{-5} = \frac{24}{x^5}$$

25 a) $f'(x) = \frac{1}{x}$

$f''(x) = -1x^{-2}$
 $f''(x) = -\frac{1}{x^2}$

$f'''(x) = 2x^{-3}$
 $f'''(x) = \frac{2}{x^3}$

$f^{(4)}(x) = -6x^{-4}$
 $f^{(4)}(x) = -\frac{6}{x^4}$

$f^{(5)}(x) = \frac{24}{x^5}$

b) $f^{(n)} = \frac{(n-1)!(-1)^{n-1}}{x^n}$

27-32

27 Inflection Pt: (2, 3)

Concave up: (2, ∞) or $x > 2$
Concave down: (-∞, 2) or $x < 2$

28 Inflection Pt: (3, 7)

Concave up: (-∞, 3)
Concave down: (3, ∞)

29 Inflection Points: (-1, 2) & (8, 6)

Concave up: (-∞, -1) & (8, ∞)
Concave down: (-1, 8)

30 Inflection Pt: (-2, 4) & (6, -1)

Concave up: (-2, 6)
Concave down: (-∞, -2) & (6, ∞)

31