Calc 5.3

#33-47 odd

(33) $f'(x) = 2x + 10$

$f''(x) = 2$ always positive $\Rightarrow$ concave up for all values in domain.

No inflection pt

(35) $f'(x) = -6x^2 + 18x + 168$

$f''(x) = -12x + 18 = 0$

$\frac{18}{12} = \frac{12x}{12}$

$\frac{3}{2} = x$

+ | − (at $\frac{3}{2}$)

inflection pt: $x = \frac{3}{2}$
concave up: $x < \frac{3}{2}$
concave down: $x > \frac{3}{2}$

(37) $f'(x) = \frac{0(x-5) - 3(1)}{(x-5)^2} = \frac{-3}{(x-5)^2} = -3(x-5)^{-2}$

$f''(x) = 6(x-5)^{-3} = \frac{6}{(x-5)^3}$ Doesn't $= 0$
DNE @ $x = 5$

− | + (at 5)

No inflection pt (vertical asymptote)
concave up: $x > 5$
concave down: $x < 5$

(39) $f'(x) = 1(x+5)^2 + (x)(2)(x+5)(1) = (x+5)(x+5+2x) = (x+5)(3x+5)$

$= 3x^2 + 15x + 5x + 25$

$f'(x) = 3x^2 + 20x + 25$

$f''(x) = 6x + 20 = 0$

$6x = -20$

$x = \frac{-20}{6} = \frac{-10}{3}$

−5, 0
− | + (at $\frac{-10}{3}$)

inflection pt: $x = \frac{-10}{3}$
concave up: $x > \frac{-10}{3}$
concave down: $x < \frac{-10}{3}$

(41) $F'(x) = 18 - 18e^{-x}(-1) = 18 + 18e^{-x}$

$F''(x) = 18e^{-x}(-1) = -18e^{-x}$

$-18e^{-x} = 0$

$\frac{-18}{e^x} = 0$

No solution — No x-value that makes F'' dne

F'' is always neg. ⇒ concave down for all x value

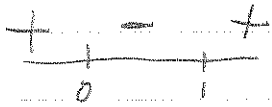
(43) $F'(x) = \frac{8}{3}x^{5/3} - \frac{20}{3}x^{2/3}$

$F''(x) = \frac{40}{9}x^{2/3} - \frac{40}{9}x^{-1/3} = \frac{40}{9}x^{-1/3}[x - 1] = 0$

$\frac{40}{9\sqrt[3]{x}} = 0$ — No solution but undefined @ $x = 0$

$x - 1 = 0$

$x = 1$



inflection pts: @ $x = 0$ & $x = 1$

concave up: $x < 0$ $x > 1$

concave down: $0 < x < 1$

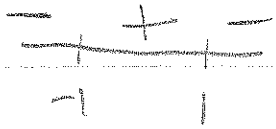
(45) $F'(x) = \frac{2x}{x^2+1}$ $F''(x) = \frac{2(x^2+1) - (2x)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$F''(x) = \frac{2-2x^2}{(x^2+1)^2} = 0 \Rightarrow 2 - 2x^2 = 0$

$2 = 2x^2$

$1 = x^2$

$\pm 1 = x$



inflection pts @ $x = 1$ & $x = -1$

concave down: $x < -1$ $x > 1$

concave up: $-1 < x < 1$

(47) $F'(x) = 2x(\text{Log}\,x) + x^2 \frac{1}{(\ln 10)x}$

$= 2x(\text{Log}\,x) + \frac{x}{\ln 10}$

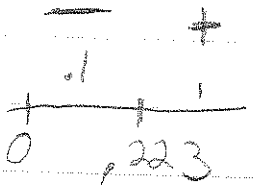
$F''(x) = 2\text{Log}\,x + (2x)\frac{1}{\ln 10 \cdot x} + \frac{1}{\ln 10}$

$= 2\text{Log}\,x + \frac{2}{\ln 10} + \frac{1}{\ln 10} = 2\text{Log}\,x + \frac{3}{\ln 10} = 0$

$\frac{2\text{Log}\,x}{2} = \frac{-3}{\ln 10 \cdot 2}$

$\text{Log}\,x = \frac{-3}{2\ln 10}$

$x = 10^{\frac{-3}{2\ln 10}} \approx .223$



inflection pt: @ $x \approx .223$

concave up: $x > .223$

concave down: $0 < x < .223$