Calculus 6.2

1) a) $y = 180 - x$

b) $P = x(180 - x) = 180x - x^2$

c) Domain: $0 \le x \le 180$

d) $\frac{dP}{dx} = 180 - 2x = 0 \Rightarrow x = 90$

e) $P(0) = 0$

$P(90) = 8100$

$P(180) = 0$

F) Maximum of $P = 8100$ $x = 90$ $y = 90$

2) $x + y = 140$ $P = x^2 + y^2$ minimized

a) $y = 140 - x$

b) $P = x^2 + (140 - x)^2$

c) $0 \le x \le 140$

d) $\frac{dP}{dx} = 2x + 2(140 - x)(-1) = 2x - 280 + 2x = 4x - 280 = 0$

$x = 70$

e) $P(0) = 19600$

$P(70) = 9800$

$P(140) = 19600$

F) Minimum of 9800 $x = 70$ $y = 70$

7 a) $R(x) = x\left(160 - \frac{x}{10}\right) = \boxed{160x - \frac{1}{10}x^2}$

b) $R'(x) = 160 - \frac{1}{5}x = 0$

$160 = \frac{1}{5}x$

$\boxed{800 = x}$

c) $R(800) = 800\left(160 - \frac{800}{10}\right) = \boxed{64{,}000}$

15 Income = weight $\left(\frac{\text{per}}{100\text{ lbs}}\right) \cdot \$\left(\frac{\text{per}}{100\text{ lbs}}\right)$

weight $= 120 + 4x$ $\qquad x = \#$ of days

$\$ = 7.50 - .15x$

Income $= (120 + 4x)(7.50 - .15x)$

$= 900 + 30x - 18x - .6x^2$

Income $= 900 + 12x - .6x^2$

$I' = 12 - 1.2x = 0$

$12 = 1.2x$

$\boxed{10 = x}$ best day to take aluminum

Income $= 900 + 120 - 60 = \boxed{\$960}$

(16) amount of material is Surface Area

Given: $V = 32\ in^3$ square base

$V = x^2 \cdot h$

$S.A._{box} = x^2 + 4xh$

$32 = x^2 h$

$\frac{32}{x^2} = h$

$S.A. = x^2 + 4x\left(\frac{32}{x^2}\right)$

$S.A. = x^2 + \frac{128}{x}$

$S.A.' = 2x - \frac{128}{x^2} = 0$

$2x = \frac{128}{x^2}$

$2x^3 = 128$

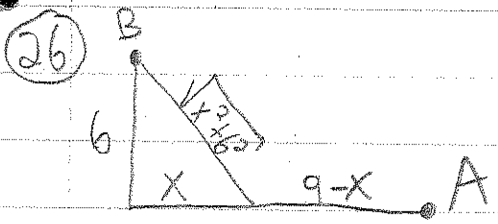
$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

$h = \frac{32}{4^2} = 2$

$\boxed{\text{Dimensions: } 4'' \times 4'' \times 2''}$

(26)



9 $\quad 0 \le x \le 9$

totalcost = cost on Land + cost underwater

cost Land = $(9-x)400$

cost water = $\left(\sqrt{x^2+6^2}\right) \cdot 500$

$C(x) = 3600 - 400x + 500(x^2+36)^{\frac{1}{2}}$

$C'(x) = -400 + 250(x^2+36)^{-1/2}(2x) = -400 + \frac{500x}{\sqrt{x^2+36}}$

$400 = \frac{500x}{\sqrt{x^2+36}}$

$400\sqrt{x^2+36} = 500x$

$160000(x^2+36) = 250000x^2$

$160000x^2 + 5760000 = 250000x^2$

$5760000 = 90000x^2$

$64 = x^2$

$x = 8$

$C(0) = \$6600$

$C(8) = \$5400$

$C(9) = 5408.33$

$\boxed{\text{1 mile from A then underwater}}$

(28) $\$ = 160x \quad : x \leq 250$

$\$ = [160 - .5(x-250)]\, x \quad : x > 25$

$= 160x - .5x^2 + 125x$

$\$ = 285x - .5x^2$

$\$' = 285 - x = 0$

$x = 285 \rightarrow \$ = 285(285) - \frac{1}{2}(285^2)$

$\$ = 40612.5$

Max \$ deal @ 285 tables

$\$ = 160(250) = \$40000$

Minimum deal at any # of tables under 250

(32) a) $p(t) = \frac{20}{1000} t^3 - \frac{1}{1000} t^4$

$p'(t) = \frac{60}{1000} t^2 - \frac{4}{1000} t^3 = 0$

$\frac{4}{1000} t^2 (15 - t) = 0$

$t = 0 \quad t = 15$

$P(0) = 0$

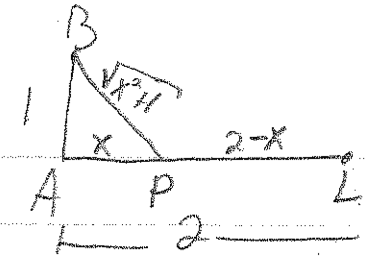
$P(15) = \frac{135}{8} \approx 16.875\%$

$P(20) = 0$

$\boxed{15 \text{ days}}$

b) 16.875%

(40) Energy = Land + $\frac{4}{3}$ water



$E = 2 - x + \frac{4}{3}\left(\sqrt{x^2+1}\right)$

$E' = -1 + \frac{4}{6}(x^2+1)^{-1/2}(2x)$

$E'(x) = -1 + \dfrac{4x}{3\sqrt{x^2+1}} = 0 \rightarrow \dfrac{4x}{3\sqrt{x^2+1}} = 1$

$4x = 3\sqrt{x^2+1}$

$16x^2 = 9(x^2+1)$

$16x^2 = 9x^2 + 9$

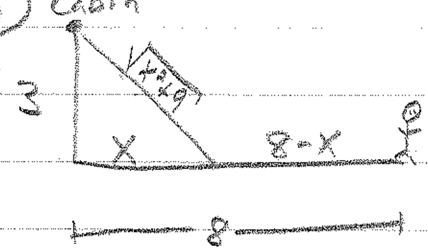
$7x^2 = 9$

$x^2 = \frac{9}{7}$

$x = \sqrt{\frac{9}{7}} \approx 1.134$

$\boxed{\text{Point P is 1.134 miles from point A.}}$

(45) Cabin



$0 \le x \le 8$

$d = rt \Longrightarrow t = \frac{d}{r}$

total time = time along river + time in woods

$t(x) = \dfrac{8-x}{5} + \dfrac{\sqrt{x^2+9}}{2} = \frac{8}{5} - \frac{1}{5}x + \frac{1}{2}\sqrt{x^2+9}$

$t'(x) = -\frac{1}{5} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(x^2+9)^{-1/2}(2x) = -\frac{1}{5} + \dfrac{x}{2\sqrt{x^2+9}} = 0$

$\frac{1}{5} = \dfrac{x}{2\sqrt{x^2+9}} \rightarrow \left(2\sqrt{x^2+9}\right)^2 = (5x)^2$

$4(x^2+9) = 25x^2$

$4x^2 + 36 = 25x^2$

$36 = 21x^2$

$\sqrt{\frac{36}{21}} = x \approx 1.309$

$t(0) = 3.1$

$t(1.309) = 2.97$ * min. time

$t(8) = 4.27$

$\boxed{\text{He should go 6.691 miles up river}}$