Calculus 6.4

#1-4, 5-15 odd

① $12x + 10y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-12x}{10y}$$

② $14x - 8y\frac{dy}{dx} = 0$

$$\frac{14x}{8y} = \frac{dy}{dx}$$

③ $16x - \left(10y + 10x\frac{dy}{dx}\right) + 6y\frac{dy}{dx} = 0$

$16x - 10y - 10x\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$

$-10x\frac{dy}{dx} + 6y\frac{dy}{dx} = 10y - 16x$

$\frac{dy}{dx}(6y - 10x) = 10y - 16x$

$$\frac{dy}{dx} = \frac{10y - 16x}{6y - 10x} = \frac{5y - 8x}{3y - 5x}$$

④ $14x = 10y\frac{dy}{dx} + \left(4y + 4x\frac{dy}{dx}\right)$

$14x = 10y\frac{dy}{dx} + 4y + 4x\frac{dy}{dx}$

$14x - 4y = \frac{dy}{dx}(10y + 4x)$

$$\frac{dy}{dx} = \frac{14x - 4y}{10y + 4x} = \frac{7x - 2y}{5y + 2x}$$

(5) $15x^2 = 6y\frac{dy}{dx} + 4\frac{dy}{dx}$

$15x^2 = (6y+4)\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{15x^2}{6y+4}}$$

(7) $6x = -\frac{\frac{dy}{dx}(2+y) - (3-y)\left(\frac{dy}{dx}\right)}{(2+y)^2}$

$6x(2+y)^2 = \frac{dy}{dx}(-2-y-2+y)$

$6x(2+y)^2 = \frac{dy}{dx}(-4) \rightarrow \boxed{\frac{dy}{dx} = \frac{6x(2+y)^2}{-4}}$

(6) $2x^{\frac{1}{2}} + 4y^{\frac{1}{2}} = 5y$

$x^{-\frac{1}{2}} + 2y^{-\frac{1}{2}}\frac{dy}{dx} = 5\frac{dy}{dx}$

$x^{-\frac{1}{2}} = 5\frac{dy}{dx} - 2y^{-\frac{1}{2}}\frac{dy}{dx}$

$x^{-\frac{1}{2}} = \frac{dy}{dx}\left(5 - 2y^{-\frac{1}{2}}\right)$

$\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}}{5 - 2y^{-\frac{1}{2}}} = \boxed{\frac{1}{\sqrt{x}\left(5 - \frac{2}{\sqrt{y}}\right)}}$

(11) $4x^3y^3 + x^4 3y^2\frac{dy}{dx} + 6x^{1/2} = 9y^{1/2}\frac{dy}{dx}$

$4x^3y^3 + 6\sqrt{x} = 9\sqrt{y}\,\frac{dy}{dx} - x^4 3y^2\frac{dy}{dx}$

$4x^3y^3 + 6\sqrt{x} = \frac{dy}{dx}(9\sqrt{y} - 3x^4y^2)$

$$\frac{dy}{dx} = \frac{4x^3y^3 + 6\sqrt{x}}{9\sqrt{y} - 3x^4y^2}$$

(13) $e^{x^2y}\left(2xy + x^2\frac{dy}{dx}\right) = 5 + 4\frac{dy}{dx}$

$2xye^{x^2y} + x^2e^{x^2y}\frac{dy}{dx} = 5 + 4\frac{dy}{dx}$

$2xye^{x^2y} - 5 = x^2e^{x^2y}\frac{dy}{dx} + 4\frac{dy}{dx}$

$2xye^{x^2y} - 5 = \left(x^2e^{x^2y} + 4\right)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2xye^{x^2y} - 5}{x^2e^{x^2y} + 4}$$

(15) $1 + \left(\frac{1}{y}\right)\frac{dy}{dx} = 2xy^3 + x^2 3y^2\frac{dy}{dx}$

$1 - 2xy^3 = 3x^2y^2\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx}$

$1 - 2xy^3 = \frac{dy}{dx}\left(3x^2y^2 - \frac{1}{y}\right)$

$\frac{dy}{dx} = \frac{1 - 2xy^3}{3x^2y^2 - \frac{1}{y}} \cdot \frac{y}{y} = \frac{y - 2xy^4}{3x^2y^3 - 1}$