Calculus Review answers ch. 6

① $F'(x) = -3x^2 + 12x = 0$

$-3x(x-4) = 0$

$x = 0 \quad x = 4$

$F(-1) = 8$

$F(0) = 1$

$F(4) = 33$ – absolute Max

$F(6) = 1$

absolute min @ $x = 0$ & 6

② $F'(x) = 12x^2 - 18x = 0$

$6x(2x-3) = 0$

$x = 0 \quad x = 3/2$

$F(-1) = -16$ ← Abs. Min

$F(0) = -3$ ← abs. Max

$F(\frac{3}{2}) = -9.75$

$F(2) = -7$

③ $F'(x) = 3x^2 + 4x - 15 = 0$

$(3x-5)(x+3) = 0$

$x = \frac{5}{3} \quad x = -3$

$F(-4) = 31$

$F(-3) = 39$ ← abs. Max

$F(\frac{5}{3}) = -11.81$ ← abs. Min

$F(2) = -1$

④ $F'(x) = -6x^2 - 4x + 2 = 0$

$-2(3x^2 + 2x - 1) = 0$

$-2(3x-1)(x+1) = 0$

$x = \frac{1}{3} \quad x = -1$

$F(-3) = 29$ ← abs. Max

$F(-1) = -3$

$F(\frac{1}{3}) = -.629$

$F(1) = -3$ ← Abs. Min

(10) $\left(2xy^3 + x^2 3y^2 \frac{dy}{dx}\right) + \left(4y + 4x\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(3x^2y^2 + 4x) = -2xy^3 - 4y$

$$\boxed{\frac{dy}{dx} = \frac{-2xy^3 - 4y}{3x^2y^3 + 4x}}$$

(11) $2x - 8y\frac{dy}{dx} = 9x^2y^4 + 3x^3\left(4y^3\frac{dy}{dx}\right)$

$2x - 9x^2y^4 = \left(12x^3y^3 + 8y\right)\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{2x - 9x^2y^4}{12x^3y^3 + 8y}}$$

(12) $\frac{9}{2\sqrt{x}} + 12y^2\frac{dy}{dx} = \frac{1}{\sqrt{y}}\frac{dy}{dx}$

$\left(12y^2 - \frac{1}{\sqrt{y}}\right)\frac{dy}{dx} = -\frac{9}{2\sqrt{x}}$

$$\frac{dy}{dx} = \frac{-9}{2\sqrt{x}} \cdot \frac{1}{\left(12y^2 - \frac{1}{\sqrt{y}}\right)} = \boxed{\frac{-9}{24y^2\sqrt{x} - \frac{2\sqrt{y}}{\sqrt{y}}}}$$

(13) $\frac{2}{2\sqrt{y-1}}\frac{dy}{dx} = \frac{6}{x^{\frac{1}{3}}} + \frac{dy}{dx}$

$\left(\frac{1}{\sqrt{y-1}} + 1\right)\frac{dy}{dx} = \frac{6}{\sqrt[3]{x}}$

$$\frac{dy}{dx} = \frac{6}{\sqrt[3]{x}\left(\frac{1}{\sqrt{y-1}} + 1\right)} = \boxed{\frac{6}{\frac{\sqrt[3]{x}}{\sqrt{y-1}} + \sqrt[3]{x}}}$$

(14) $\dfrac{\left(1+2\frac{dy}{dx}\right)(x-3y)-(x+2y)\left(1-3\frac{dy}{dx}\right)}{(x-3y)^2}=\dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx}$

$\dfrac{x+2x\frac{dy}{dx}-3y-6y\frac{dy}{dx}-x+3x\frac{dy}{dx}-2y+6y\frac{dy}{dx}}{(x-3y)^2}=\dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx}$

$x-3y-x-2y+2x\dfrac{dy}{dx}-6y\dfrac{dy}{dx}+3x\dfrac{dy}{dx}+6y\dfrac{dy}{dx}=\dfrac{(x-3y)^2}{2\sqrt{y}}\dfrac{dy}{dx}$

$-5y+5x\dfrac{dy}{dx}=\dfrac{(x-3y)^2}{2\sqrt{y}}\dfrac{dy}{dx}$

$\dfrac{dy}{dx}\left(5x-\dfrac{(x-3y)^2}{2\sqrt{y}}\right)=5y$

$\dfrac{dy}{dx}=\dfrac{5y}{5x-\frac{(x-3y)^2}{2\sqrt{y}}}=\boxed{\dfrac{10y\sqrt{y}}{10x\sqrt{y}-(x-3y)^2}}$

(15) $\dfrac{5(2-3y)-(6+5x)\left(-3\frac{dy}{dx}\right)}{(2-3y)^2}=\dfrac{-1}{5x^2}$ $\quad \frac{1}{5}x^{-1}$

$10-15y+18\dfrac{dy}{dx}+15x\dfrac{dy}{dx}=\dfrac{-(2-3y)^2}{5x^2}$

$\dfrac{dy}{dx}(18+15x)=\dfrac{-(2-3y)^2}{5x^2}-10+15y$

$\dfrac{dy}{dx}=\dfrac{\frac{-(2-3y)^2}{5x^2}-10+15y}{18+15x}=\boxed{\dfrac{-(2-3y)^2-50x^2+75x^2y}{90x^2+75x^3}}$

(16) $\frac{1+\frac{dy}{dx}}{x+y} = 2x + 3y^2\frac{dy}{dx}$

$1+\frac{dy}{dx} = \left(2x+3y^2\frac{dy}{dx}\right)(x+y)$

$1+\frac{dy}{dx} = 2x^2 + 3xy^2\frac{dy}{dx} + 2xy + 3y^3\frac{dy}{dx}$

$1-2x^2-2xy = \left(3xy^2+3y^3-1\right)\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{1-2x^2-2xy}{3xy^2+3y^3-1}}$

(17) $\frac{y+x\frac{dy}{dx}}{xy+1} = 2y^3 + 2x3y^2\frac{dy}{dx}$

$y+x\frac{dy}{dx} = \left(2y^3+6xy^2\frac{dy}{dx}\right)(xy+1)$

$y+x\frac{dy}{dx} = 2xy^4 + 6x^2y^3\frac{dy}{dx} + 2y^3 + 6xy^2\frac{dy}{dx}$

$y-2xy^4-2y^3 = \left(6x^2y^3+6xy^2-x\right)\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{y-2xy^4-2y^3}{6x^2y^3+6xy^2-x}}$

(18) $\frac{1}{2}(2x)^{-1/2}(2) - 4x\frac{dy}{dx} - 4y = 0$

$\frac{1}{2}(4)^{-1/2}(2) - 4(2)\frac{dy}{dx} - 12 = 0$

$\frac{1}{2} - 8\frac{dy}{dx} = 12$

$-8\frac{dy}{dx} = 11.5$

$\frac{dy}{dx} = \frac{11.5}{-8} = \frac{-23}{16} = m$

$y-3 = \frac{-23}{16}(x-2)$

$y = \frac{-23}{16}x + \frac{23}{8} + \frac{24}{8}$

$\boxed{y = \frac{-23}{16}x + \frac{47}{8}}$

(21) $\frac{dy}{dt} = 24x^2\frac{dx}{dt} - 14x\frac{dx}{dt}$

$\frac{dy}{dt} = 24(4)(4) - 14(2)(4) = 384 - 112 = \boxed{272}$

(22) $\frac{dy}{dt} = \frac{\left(-4\frac{dx}{dt}\right)(3+2x) - (9-4x)\left(2\frac{dx}{dt}\right)}{(3+2x)^2}$

$\frac{dy}{dt} = \frac{-4(-1)(-3) - (21)(-2)}{9} = \frac{-12+42}{9} = \boxed{\frac{30}{9}} = \boxed{\frac{10}{3}}$

(23) $\frac{dy}{dt} = \frac{\left(\frac{1}{2\sqrt{x}}\frac{dx}{dt}\right)(1-\sqrt{x}) - (1+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\frac{dx}{dt}\right)}{(1-\sqrt{x})^2} = \frac{\left(\frac{1}{4}\right)(-4)(1-2) - (3)\left(\frac{1}{4}\right)(-4)}{(1-2)^2}$

$= \frac{1-3}{1} = \boxed{-2}$

(24) $\left(2x\frac{dx}{dt} + 5\frac{dy}{dt}\right)(x-2y) - (x^2+5y)\left(\frac{dx}{dt} - 2\frac{dy}{dt}\right) = 0$

$\left(2(2)(1) + 5\frac{dy}{dt}\right)(2-0) - (4+0)\left(1 - 2\frac{dy}{dt}\right) = 0$

$8 + 10\frac{dy}{dt} - 4 + 8\frac{dy}{dt} = 0$

$18\frac{dy}{dt} = -4 \qquad \frac{dy}{dt} = \frac{-4}{18} = \boxed{\frac{-2}{9}}$

(25) $\frac{dy}{dt} = \frac{dx}{dt}(e^{3x}) + x(e^{3x})\left(3\frac{dx}{dt}\right)$

$\frac{dy}{dt} = (-2)(e^3) + (1)(e^3)(3)(-2) = -2e^3 - 6e^3 = \boxed{-8e^3}$

(26) $y = (e^{x^2}+1)^{-1}$

$\frac{dy}{dt} = (-1)(e^{x^2}+1)^{-2}(e^{x^2})(2x\frac{dx}{dt})$

$= (-1)(e^1+1)^{-2}(e^1)(2)(3)$

$\frac{dy}{dt} = \frac{-6e}{(e+1)^2}$

(28) $dy = (-2x+3x^3)\,dx$

$dy = (2+3)(.02)$

$dy = .1$

(29) $dy = \frac{3(2x+1)-2(3x-7)}{(2x+1)^2}\,dx$

$dy = \frac{15+2}{25}\cdot(.003)$

$= \frac{17}{25}\cdot(.003) \approx .00204$

(32) a) $P'(x) = -3x^2+20x-12 = 0$

$-1(3x^2-20x+12) = 0$

$-(3x-2)(x-6)$

$x = \frac{2}{3} \quad x = 6$

$P(0) = 0$

$P(\frac{2}{3}) = -3.85$

$P(6) = 72$

600 boxes

b) $P(6) = 72$

$720

(34) $V = 40\ in^3$ $\quad V = \pi r^2 h$ $\quad cost_{\text{top \& bottom}} = 4¢/in^2$

$40 = \pi r^2 h$

$\frac{40}{\pi r^2} = h$ $\quad cost_{side} = 3¢/in^2$

$r = ?$

$h = ?$

$S.A. = 2(\pi r^2) + (2\pi r)h$

$S.A. = 2\pi r^2 + 2\pi r\left(\frac{40}{\pi r^2}\right)$

$S.A. = 2\pi r^2 + \frac{80}{r}$

$S' = 4\pi r + -80 r^{-2} = 0$

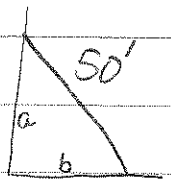
$4\pi r^3 - 80 = 0$

$r^3 = \frac{80}{4\pi} \approx 6.366$

$r \approx 1.85$

$h = \frac{40}{\pi(1.85)^2} = 3.71 = h$

(45)



$\frac{da}{dt} = -2\ \frac{ft}{min}$ $\quad \frac{db}{dt} = ?$ $\quad b = 30$ $\quad a = 40$ $\quad a^2 + 30^2 = 50^2$

$a^2 + b^2 = 50^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$80(-2) + 60\left(\frac{db}{dt}\right) = 0$

$60\frac{db}{dt} = 160$

$\frac{db}{dt} = \frac{160}{60} = \frac{8}{3}\ \frac{ft}{min}$

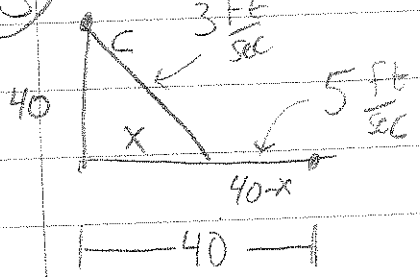
(48) $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 \, dr$

$dV = 4\pi(12)(.02)$

$\therefore dV = 1.28\pi \approx \boxed{4.02 \text{ in}^3}$

(53)



$t = \frac{d}{r}$

$t_{shore} = \frac{40-x}{5}$

$t_{water} = \frac{\sqrt{x^2+40^2}}{3}$

$x^2 + 40^2 = c^2$

$c = \sqrt{x^2+40^2}$

$t_{total} = \frac{2}{5} - \frac{1}{5}x + \frac{1}{3}(x^2+1600)^{1/2}$

$t' = \frac{-1}{5} + \frac{1}{6}(x^2+1600)^{-1/2}(2x) = 0$

$\frac{x}{3\sqrt{x^2+1600}} = 0$

$x = 0 \Rightarrow$ he should walk all 40' on beach then paddle straight out to spot.