Calculus 7.2

(2) a) $u = 3x^2 - 5$ b) $u = 1 - x$ c) $u = 2x^3 + 1$
d) $u = x^4$

(3) $u = 2x + 3$
$du = 2\,dx$
$2du = 4\,dx$

$\int 2u^4\,du = \frac{2u^5}{5} + C$

$= \boxed{\frac{2}{5}(2x+3)^5 + C}$

(4) $u = -4t + 1$
$du = -4\,dt$
$-\frac{1}{4}du = dt$

$\int -\frac{1}{4}u^3\,du$

$-\frac{1}{4}\frac{u^4}{4} + C = \boxed{-\frac{1}{16}(-4t+1)^4 + C}$

(5) $\int 2(2m+1)^3\,dm$

$u = 2m + 1$
$du = 2\,dm$

$\int u^{-3}\,du$

$= \frac{u^{-2}}{-2} + C = \frac{(2m+1)^{-2}}{-2} + C = \boxed{\frac{-1}{2(2m+1)^2} + C}$

(6) $\int 3(3u-5)^{-1/2}\,du$

$\int x^{-1/2}\,dx$

$= \frac{x^{1/2}}{\frac{1}{2}} + C$

$= \boxed{2\sqrt{3u-5} + C}$

$x = 3u - 5$
$dx = 3\,du$

⑦ $\int (2x+2)(x^2+2x-4)^{-4}\,dx$

$u = x^2+2x-4$

$du = 2x+2\,dx$

$= \int u^{-4}\,du$

$= \frac{u^{-3}}{-3} + C$

$= \boxed{\frac{-1}{3(x^2+2x-4)^3} + C}$

⑧ $\int 6x^2(2x^3+7)^{-3/2}\,dx$

$u = 2x^3+7$

$du = 6x^2\,dx$

$\int u^{-3/2}\,du$

$\frac{u^{-1/2}}{-\frac{1}{2}} + C$

$\boxed{\frac{-2}{\sqrt{2x^3+7}} + C}$

⑨ $\int z(4z^2-5)^{1/2}\,dz$

$u = 4z^2-5$

$du = 8z\,dz$

$-\frac{1}{8}du = z\,dz$

$\int \frac{1}{8} u^{1/2}\,du$

$\frac{1}{8}\frac{u^{3/2}}{3/2} + C$

$\frac{2}{3}\frac{1}{8}$

$\boxed{\frac{1}{12}(4z^2-5)^{3/2} + C}$

⑩ $\int r(5r^2+2)^{1/2}\,dr$

$u = 5r^2+2$

$du = 10r\,dr$

$\frac{1}{10}du = r\,dr$

$\int \frac{1}{10} u^{1/2}\,du$

$\frac{1}{10}\frac{u^{3/2}}{3/2} + C$

$\boxed{\frac{1}{15}(5r^2+2)^{3/2} + C}$

(11) $u = 2x^3$

$du = 6x^2\,dx$

$\frac{1}{2}du = 3x^2\,dx$

$\int \frac{1}{2} e^u\,du$

$= \frac{1}{2}e^u + C$

$= \boxed{\frac{1}{2}e^{2x^3} + C}$

(12) $u = -r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$\int -\frac{1}{2} e^u\,du$

$-\frac{1}{2}e^u + C$ $= \boxed{-\frac{1}{2}e^{-r^2} + C}$

(13) $u = 2t - t^2$

$du = 2 - 2t\,dt$

$\frac{1}{2}du = (1-t)\,dt$

$\int \frac{1}{2} e^u\,du$

$= \frac{1}{2}e^u + C$

$= \boxed{\frac{1}{2}e^{2t-t^2} + C}$

(14) $u = x^3 - 3x$

$du = 3x^2 - 3\,dx$

$\frac{1}{3}du = (x^2 - 1)\,dx$

$\int \frac{1}{3} e^u\,du$

$= \frac{1}{3}e^u + C$

$= \boxed{\frac{1}{3}e^{x^3-3x} + C}$

(15) $u = \frac{1}{z} = z^{-1}$

$du = -z^{-2}\,dz$

$du = \frac{-1}{z^2}\,dz$

$-du = \frac{1}{z^2}\,dz$

$\int \frac{e^{1/z}}{z^2}\,dz = \int e^u \frac{1}{z^2}\,dz$

$= \int -e^u\,du = -e^u + C$

$= \boxed{-e^{1/z} + C}$

(16) $\int e^{\sqrt{y}}\left(\frac{1}{2\sqrt{y}}\right) dy$ $\quad u = \sqrt{y}$

$du = \frac{1}{2\sqrt{y}}\, dy$

$\int e^u\, du$

$e^u + C$

$\boxed{e^{\sqrt{y}} + C}$

(17) $u = x^4 + 4x^2 + 7$ $\quad \int \frac{1}{4} u^4\, du$

$du = 4x^3 + 8x\, dx$ $\quad = \frac{1}{4}\,\frac{u^5}{5} + C$

$\frac{1}{4}du = (x^3 + 2x)\, dx$

$= \frac{(x^4 + 4x^2 + 7)^5}{20} + C$

(18) $u = t^3 + 6t + 3$ $\quad \int \frac{1}{3}\,\frac{1}{u}\, du$

$du = 3t^2 + 6\, dt$ $\quad = \frac{1}{3} Ln|u| + C$

$\frac{1}{3}du = t^2 + 2\, dt$

$= \boxed{\frac{1}{3} Ln|t^3 + 6t + 3| + C}$

(19) $u = x^2 + x$ $\quad \int \frac{1}{u^3}\, du = \int u^{-3}\, du = \frac{u^{-2}}{-2} + C$

$du = 2x + 1\, dx$

$= \boxed{\frac{-1}{2(x^2 + x)^2} + C}$

(20) $\int (y^2+y)(2y^3+3y^2+1)^{2/3}\,dy$ $\quad u = 2y^3+3y^2+1$

$du = 6y^2+6y\,dy$

$\frac{1}{6}du = y^2+y\,dy$

$\int \frac{1}{6} u^{-2/3}\,du$

$= \frac{1}{6}\,\frac{u^{1/3}}{\frac{1}{3}} + C = \boxed{\frac{1}{2}\sqrt[3]{2y^3+3y^2+1} + C}$

(21) $\int p(p+1)^5\,dp$ $\quad u = p+1 \quad p = u-1$

$du = dp$

$\int (u-1)(u^5)\,du$

$\int u^6 - u^5\,du = \frac{u^7}{7} - \frac{u^6}{6} + C$

$= \boxed{\frac{(p+1)^7}{7} - \frac{(p+1)^6}{6} + C}$

(22) $\int 4r(8-r)^{1/2}\,dr$ $\quad u = 8-r \quad r = 8-u$

$\int (32-4u)\,u^{1/2}\,du$ $\quad du = -dr \quad 4r = 32-4u$

$-du = dr$

$\int (-32u^{1/2} + 4u^{3/2})\,du$

$= \frac{-32\,u^{3/2}}{3/2} + \frac{4\,u^{5/2}}{5/2} + C = \boxed{\frac{-64}{3}(8-r)^{3/2} + \frac{8}{5}(8-r)^{5/2} + C}$

(23) $\int \frac{v}{\sqrt{v-1}}\,dv$ $\quad a = v-1 \quad da = dv$

$a+1 = v$

$\int v(v-1)^{-1/2}\,dv$

$\int (a+1)\,a^{-1/2}\,da = \int (a^{1/2} + a^{-1/2})\,da$

$= \frac{a^{3/2}}{3/2} + \frac{a^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{3}(v-1)^{3/2} + 2(v-1)^{1/2} + C}$

(24) $\int (2x)(x+5)^{-6}\,dx \quad u = x+5 \quad x = u-5 \quad 2x = 2u-10$

$du = dx$

$\int (2u-10)(u^{-6})\,du$

$\int (2u^{-5} - 10u^{-6})\,du = \frac{2u^{-4}}{-4} - \frac{10u^{-5}}{-5} + C$

$= \boxed{\frac{-1}{2(x+5)^4} + \frac{2}{(x+5)^5} + C}$