(25) $u = x^2 + 12x$

$du = 2x + 12\,dx$

$\frac{1}{2}du = x + 6\,dx$

$\int \frac{1}{2} u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{3}(x^2 + 12x)^{3/2} + C$

(26) $u = x^2 - 6x$

$du = 2x - 6\,dx$

$\frac{1}{2}du = x - 3\,dx$

$\int \frac{1}{2} u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$\frac{1}{3}(x^2 - 6x)^{3/2} + C$

(27) $\int t(t^2+1)^{-1}\,dt$

$u = t^2 + 1$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$= \int \frac{1}{2} u^{-1}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|t^2+1| + C$

(28) $\int (-4x)(x^2+3)^{-1}\,dx$

$u = x^2 + 3$

$du = 2x\,dx$

$-2du = -4x\,dx$

$\int -2u^{-1}\,du$

$= -2\ln|u| + C$

$= -2\ln|x^2+3| + C$

(29) $\int (1+3\ln x)^2 \left(\frac{1}{x}\right) dx$

$u = 1 + 3\ln x$

$du = 3\frac{1}{x}\,dx$

$\frac{1}{3}du = \frac{1}{x}\,dx$

$= \int \frac{1}{3} u^2 \, du$

$= \frac{1}{3}\frac{u^3}{3} + C$

$= \boxed{\frac{(1+3\ln x)^3}{9} + C}$

(30) $\int (2+\ln x)^{1/2} \left(\frac{1}{x}\right) dx$

$u = 2 + \ln x$

$du = \frac{1}{x}\,dx$

$\int u^{1/2}\, du$

$= \frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{3}(2+\ln x)^{3/2} + C}$

(31) $\int e^{2x}(e^{2x}+5)^{-1}\, dx$

$u = e^{2x} + 5$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

$\int \frac{1}{2} u^{-1}\, du$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln|e^{2x}+5| + C}$

(32) $\int \left(\frac{1}{x}\right)(\ln x)^{-1}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int u^{-1}\,du$

$= \ln|u| + C$

$= \boxed{\ln|\ln|x|| + C}$

(33) $\int (\log x)\left(\frac{1}{x}\right)dx$

$u = \log x$

$du = \frac{1}{x \ln 10}\,dx$

$\ln 10 \int u\,du$

$= (\ln 10)\,\frac{u^2}{2} + C$

$(\ln 10)\,du = \frac{1}{x}\,dx$

$= \boxed{\frac{\ln 10}{2}\,(\log x)^2 + C}$

(34) $u = \log_2(5x+1)$

$\int (\log_2 5x+1)^2 \left(\frac{1}{5x+1}\right)dx$

$du = \frac{5}{\ln 2\,(5x+1)}\,dx$

$\int u^2 \left(\frac{\ln 2}{5}\right)du$

$du = \frac{5}{\ln 2}\,\frac{1}{5x+1}\,dx$

$= \frac{\ln 2}{5}\,\frac{u^3}{3} + C$

$\frac{\ln 2}{5}\,du = \frac{1}{5x+1}\,dx$

$= \boxed{\left(\frac{\ln 2}{5}\right)\frac{(\log_2(5x+1))^3}{3} + C}$

(35) $u = 3x^2 + 1$

$du = 6x\,dx$

$\frac{1}{6}du = x\,dx$

$\int \frac{1}{6} \cdot 8^u\,du$

$= \frac{1}{6}\,\frac{8^u}{\ln 8} + C$

$= \boxed{\frac{8^{3x^2+1}}{6 \ln 8} + C}$

(36) $u = 5\sqrt{x} + 2$

$du = \frac{5}{2\sqrt{x}}\,dx$

$\frac{2}{5}du = \frac{1}{\sqrt{x}}\,dx$

$\int (10^{5\sqrt{x}+2})\left(\frac{1}{\sqrt{x}}\right)dx$

$= \int \frac{2}{5} 10^u\,du$

$= \frac{2}{5}\,\frac{10^u}{\ln 10} + C$

$= \boxed{\frac{2 \cdot 10^{5\sqrt{x}+2}}{5 \ln 10} + C}$