Calculus Review - Definite Integral

Key

Evaluate

① $\int_{-2}^{2}(x^3-1)\,dx = \frac{1}{4}x^4 - x\Big|_{-2}^{2} = \left(\cancel{\tfrac{1}{4}}(\cancel{16})-2\right)-\left(\cancel{\tfrac{1}{4}}(\cancel{16})+2\right) = \boxed{-4}$

② $\int_{-1}^{2}\left(\frac{2}{x^3}+5x\right)dx$ $\quad 2x^{-3}$

$= \frac{2x^{-2}}{-2} + \frac{5}{2}x^2 = \frac{-1}{x^2} + \frac{5x^2}{2}\Big|_{-1}^{2} = \left(\frac{-1}{4}+10\right)-\left(-1+\frac{5}{2}\right) = \boxed{8.25}$

③ $\int_0^3 x\left(\sqrt[3]{x}-2\right)dx$

$\int (x^{4/3}-2x)\,dx$

$\frac{3}{7}x^{7/3} - x^2\Big|_0^3 = \left(\frac{3}{7}\left(3^{7/3}\right)-9\right)-0 = \boxed{\frac{3}{7}\,3^{7/3}-9 \approx -3.44}$

④ $\int_0^1 2x(x^2+1)^2\,dx = \int u^2\,du = \frac{1}{3}u^3 = \frac{1}{3}(x^2+1)^3\Big|_0^1 = \left(\frac{1}{3}(8)-\frac{1}{3}\right) = \boxed{\frac{7}{3}}$

$u = x^2+1$

$du = 2x\,dx$

⑤ $\int_0^4 \frac{dx}{\sqrt{2x+1}} = \int \frac{1}{2}u^{-1/2}\,du$

$u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}(2)u^{1/2}$

$(2x+1)^{1/2}\Big|_0^4 = \sqrt{9}-\sqrt{1} = \boxed{2}$

Find the total area between the function and x-axis

⑥ $f(x) = x^2 + 3x - 4$ $\quad [-1, 2]$

$0 = (x+4)(x-1)$

$x = -4 \quad x = 1$

$\int_{-1}^{1} x^2 + 3x - 4 \, dx + \int_{1}^{2} x^2 + 3x - 4 \, dx$

$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 4x \Big|_{-1}^{1} + \frac{1}{3}x^3 + \frac{3}{2}x^2 - 4x \Big|_{1}^{2} = \left[\cancel{\left(\frac{1}{3} + \frac{3}{2} - 4\right)} - \left(-\frac{1}{3} + \frac{3}{2} + 4\right)\right] + \left[\left(\frac{8}{3} + 6 - 8\right) - \cancel{\left(\frac{1}{3} + \frac{3}{2} - 4\right)}\right]$

$\left[\frac{2}{3} - 8\right]$

$-7\frac{1}{3}$

$-\frac{22}{3}$

$+ \frac{7}{3} + 2 - \frac{3}{2}$

$\frac{13}{3} - \frac{3}{2} = \frac{26}{6} - \frac{9}{6} = \frac{17}{6}$

$\frac{22}{3} + \frac{17}{6}$

$\frac{44}{6} + \frac{17}{6} = \boxed{\frac{61}{6}}$

⑦ $f(x) = 4x - x^2$ $\quad [0, 5]$

$0 = 4x - x^2$

$0 = x(4 - x)$

$x = 0, 4$

$\int_{0}^{4} 4x - x^2 \, dx + \int_{4}^{5} 4x - x^2 \, dx$

$2x^2 - \frac{1}{3}x^3 \Big|_{0}^{4} + 2x^2 - \frac{1}{3}x^3 \Big|_{4}^{5}$

$\left[\left(32 - \frac{64}{3}\right) - 0\right] + \left[\left(50 - \frac{125}{3}\right) - \left(32 - \frac{64}{3}\right)\right]$

$10.66 \quad + \left[18 - \frac{189}{3}\right]$

$|-45|$

$10.66 + 45$

$\boxed{55.66}$