7.5 Area between two Curves

① $\int_{-2}^{1} 2x^2+5\,dx$

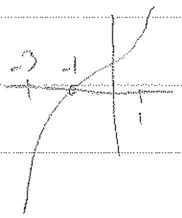
$\frac{2}{3}x^3+5x\Big|_{-2}^{1} = \left(\frac{2}{3}+5\right)-\left(\frac{-16}{3}-10\right) = \frac{18}{3}+15 = 21$

② $\int_{1}^{2} 3x^3+2\,dx = \frac{3}{4}x^4+2x\Big|_{1}^{2} = (12+4)-\left(\frac{3}{4}+2\right) = 16-2-\frac{3}{4} = 13\frac{1}{4} = \frac{53}{4}$

③ $x^3+1=0$

$x^3=-1$

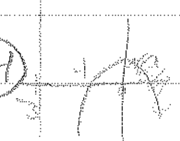
$x=-1$



$\int_{-3}^{-1} -x^3-1\,dx + \int_{-1}^{1} x^3+1\,dx = -\frac{1}{4}x^4-x\Big|_{-3}^{-1} + \frac{1}{4}x^4+x\Big|_{-1}^{1}$

$= \left(-\frac{1}{4}+1\right)-\left(-\frac{81}{4}+3\right) + \left(\frac{1}{4}+1\right)-\left(\frac{1}{4}-1\right)$

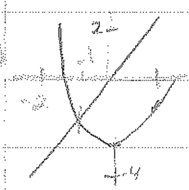
$= \frac{80}{4}+1-3 + 2 = 20$

④ 

$\int_{-3}^{-1} -1+x^2\,dx + \int_{-1}^{0} 1-x^2\,dx = -x+\frac{1}{3}x^3\Big|_{-3}^{-1} + x-\frac{1}{3}x^3\Big|_{-1}^{0}$

$= \left(1-\frac{1}{3}\right)-(3-9) + (0-0)-\left(-1+\frac{1}{3}\right)$

$\frac{2}{3}+6+1-\frac{1}{3} = 7\frac{1}{3} \quad \frac{22}{3}$

⑤ 

$x^2-3=2x$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$\cancel{x=3}\quad x=-1$

$\int_{-2}^{-1} x^2-3-2x\,dx + \int_{-1}^{1} 2x-x^2+3\,dx$

$\frac{1}{3}x^3-3x-x^2\Big|_{-2}^{-1} + x^2-\frac{1}{3}x^3+3x\Big|_{-1}^{1}$

$\left(-\frac{1}{3}+3-1\right)-\left(-\frac{8}{3}+6-4\right)+\left(1-\frac{1}{3}+3\right)-\left(1+\frac{1}{3}-3\right)$

$\frac{7}{3}$

$-\frac{2}{3}+6$

$\frac{7}{3} + -\frac{2}{3}+6 = 6\frac{5}{3} \quad \frac{23}{3}$

(6) $5x = 3x+10 \qquad \int_0^5 \overset{-2x+10}{3x+10-5x}\,dx + \int_5^6 \overset{2x-10}{5x-3x-10}\,dx$

$2x = 10$

$x = 5 \qquad -x^2+10x\Big|_0^5 + x^2-10x\Big|_5^6$

$= (-25+50) - 0 + (36-60) - (25-50) = 25 + 11 - 10 = 26$

(7) $x^2 - 30 = 10 - 3x \qquad \int_{-8}^{5} 10-3x-x^2+30\,dx = \int_{-8}^{5} -x^2-3x+40\,dx$

$x^2+3x-40=0$

$(x+8)(x-5)=0$

$x=-8 \quad x=5$

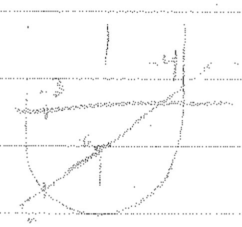
$-\frac{1}{3}x^3 - \frac{3}{2}x^2 + 40x\Big|_{-8}^{5}$

$= \left(-\frac{125}{3} - \frac{75}{2} + 200\right) - \left(\frac{512}{3} - \frac{192}{2} - 320\right)$

$= -\frac{637}{3} + \frac{117}{2} + 520$

$= 366.1\overline{6}$

(8) $x^2 - 18 = x - 6$



$\int_{-3}^{4} x-6-x^2+18\,dx = \int_{-3}^{4} -x^2+x+12\,dx$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x=4 \quad x=-3$

$= -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 12x\Big|_{-3}^{4} = \left(\frac{-64}{3} + 8 + 48\right) - \left(9 + \frac{9}{2} - 36\right)$

$-21\frac{1}{3} + 56 - 9 - \frac{9}{2} + 36$

$= 57.1\overline{6}$

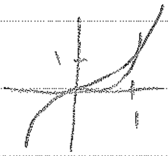
(9) $x^2 = 2x$ $\qquad \int_0^2 2x - x^2\,dx = x^2 - \frac{1}{3}x^3 \Big|_0^2 = \left(4 - \frac{8}{3}\right) = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

(10) $x^3 - x^2 = 0$



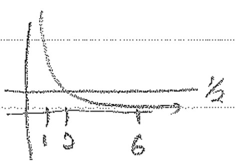
$\int_0^1 x^2 - x^3\,dx = \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1 = \frac{1}{3} - \frac{1}{4}$

$\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$

$x^2(x-1) = 0$

$x = 0 \quad x = 1$

(11)



$\int_1^2 \frac{1}{x} - \frac{1}{2}\,dx + \int_2^6 \frac{1}{2} - \frac{1}{x}\,dx$

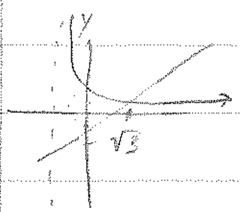
$\ln|x| - \frac{1}{2}x \Big|_1^2 + \frac{1}{2}x - \ln|x| \Big|_2^6$

$(\ln 2 - 1) - \left(0 - \frac{1}{2}\right) + (\cancel{3} - \ln 6) - (\cancel{1} - \ln 2)$

$\ln 2 - \frac{1}{2} + 2 - \ln 6 + \ln 2$

$2\ln 2 - \ln 6 + \frac{3}{2} \approx 1.095$

(12)



$\frac{1}{x+1} = \frac{x-1}{2}$ $\qquad \int_0^{\sqrt{3}} \frac{1}{x+1} - \frac{1}{2}x + \frac{1}{2}\,dx + \int_{\sqrt{3}}^4 \frac{1}{2}x - \frac{1}{2} - \frac{1}{x+1}\,dx$

$2 = x^2 - 1$

$3 = x^2$ $\qquad = \ln|x+1| - \frac{1}{4}x^2 + \frac{1}{2}x \Big|_0^{\sqrt{3}} + \frac{1}{4}x^2 - \frac{1}{2}x - \ln|x+1| \Big|_{\sqrt{3}}^4$

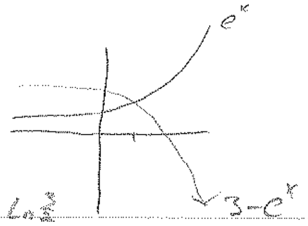
$x = \sqrt{3}$

$= \ln|\sqrt{3}+1| - \frac{3}{4} + \frac{\sqrt{3}}{2} + \left[(4 - 2 - \ln 5) - \left(\frac{3}{4} - \frac{\sqrt{3}}{2} - \ln|\sqrt{3}+1|\right)\right]$

$\cancel{\ln(\sqrt{3}+1)} - \cancel{\frac{3}{4}} + \cancel{\frac{\sqrt{3}}{2}} + 2 - \ln 5 - \cancel{\frac{3}{4}} + \cancel{\frac{\sqrt{3}}{2}} + \ln(\cancel{\sqrt{3}+1})$

$2\ln(\sqrt{3}+1) - \frac{3}{2} + \sqrt{3} + 2 - \ln 5$

$2\ln(\sqrt{3}+1) + \frac{1}{2} + \sqrt{3} - \ln 5 \approx 2.633$



$e^x = 3 - e^x$

$2e^x = 3$

$e^x = \frac{3}{2}$

$x = \ln\frac{3}{2}$

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$$\int_{-1}^{\ln\frac{3}{2}} 3 - e^x - e^x\,dx + \int_{\ln\frac{3}{2}}^{1} e^x - 3 + e^x\,dx = \int 3 - 2e^x\,dx + \int 2e^x - 3\,dx$$

$$3x - 2e^x \Big|_{-1}^{\ln\frac{3}{2}} + 2e^x - 3x \Big|_{\ln\frac{3}{2}}^{1}$$

$$(3\ln\tfrac{3}{2} - \cancel{3}) - (\cancel{-3} - \tfrac{2}{e}) + (2e - \cancel{3}) - (\cancel{3} - 3\ln\tfrac{3}{2})$$

$$6\ln\tfrac{3}{2} + \tfrac{2}{e} + 2e - 6 \approx 2.605$$

15 $2e^{2x} = e^{2x} + 1$

$$\int_{-1}^{0} e^{2x} + 1 - 2e^{2x}\,dx + \int_{0}^{2} 2e^{2x} - e^{2x} - 1\,dx$$

$e^{2x} = 1$

$x = 0$

$$\int_{-1}^{0} -e^{2x} + 1\,dx + \int_{0}^{2} e^{2x} - 1\,dx$$

$$\frac{-e^{2x}}{2} + x \Big|_{-1}^{0} + \frac{e^{2x}}{2} - x \Big|_{0}^{2}$$

$$\cancel{-\tfrac{1}{2}} - \left(\frac{-1}{2e^2} - 1\right) + \left(\frac{e^4}{2} - 2\right) - \left(\cancel{\tfrac{1}{2}}\right)$$

$$\frac{1}{2e^2} + \frac{e^4}{2} - 2 \approx 25.37$$

17 $x^3 - x^2 + x + 1 = 2x^2 - x + 1$

$$\int_0^1 x^3 - 3x^2 + 2x\,dx + \int_1^2 -x^3 + 3x^2 - 2x\,dx$$

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$$\tfrac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 + -\tfrac{1}{4}x^4 + x^3 - x^2 \Big|_1^2$$

$x(x-2)(x-1) = 0$

$$\left(\tfrac{1}{4} - \cancel{1} + \cancel{1}\right) - (0) + \cancel{(-4 + 8 - 4)} - \left(-\tfrac{1}{4} + 1 - 1\right)$$

$x = 0, 2, 1$

$$\tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}$$

(19) $x^4 + Ln(x+10) = x^3 + Ln(x+10$

$x^4 - x^3 = 0$

$x^3(x-1) = 0$

$x=0 \quad x=1$

$$\int_0^1 x^3 + Ln(x+10) - x^4 - Ln(x+10)\,dx$$

$$\int_0^1 x^3 - x^4\,dx = \frac{1}{4}x^4 - \frac{1}{5}x^5\Big|_0^1$$

$$\frac{1}{4} - \frac{1}{5} - 0$$

$$\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$$

(21) $x^{4/3} = 2x^{\frac{1}{3}}$

$x^{4/3} - 2x^{\frac{1}{3}} = 0$

$x^{\frac{1}{3}}(x-2) = 0$

$x=0 \quad x=2$

$$\int_0^2 2x^{\frac{1}{3}} - x^{4/3}\,dx = 2\cdot\frac{3}{4}x^{4/3} - \frac{3}{7}x^{7/3}\Big|_0^2$$

$$\frac{3}{2}2^{4/3} - \frac{3}{7}2^{7/3}$$

$$3.7796 - 2.15986$$

$$\approx 1.619$$

(23) $2e^{3x} = e^{3x} + e^6$

$e^{3x} = e^6$

$x=2$

$$\int_0^2 e^{3x} + e^6 - 2e^{3x}\,dx + \int_2^3 2e^{3x} - e^{3x} - e^6\,dx$$

$$\int_0^2 e^6 - e^{3x}\,dx + \int_2^3 e^{3x} - e^6\,dx$$

$$e^6x - \frac{1}{3}e^{3x}\Big|_0^2 + \frac{1}{3}e^{3x} - e^6x\Big|_2^3$$

$$\left(\cancel{2e^6} - \tfrac{1}{3}\cancel{e^6}\right) - \left(0 - \tfrac{1}{3}\right) + \left(\frac{e^9}{3} - \cancel{3e^6}\right) - \left(\cancel{\frac{e^6}{3}} - \cancel{2e^6}\right)$$

$$\frac{e^9}{3} + \frac{e^6}{3} + \frac{1}{3} \approx 2836$$

(18) $2x^3 + \cancel{x^2} + x + 5 = x^3 + \cancel{x^2} + 2x + 5$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0 \quad \pm 1$

$$\int_{-1}^{0} x^3 - x \, dx + \int_{0}^{1} -x^3 + x \, dx$$

$$\frac{1}{4}x^4 - \frac{1}{2}x^2 \Big/_{-1}^{0} + -\frac{1}{4}x^4 + \frac{1}{2}x^2 \Big/_{0}^{1}$$

$$0 - \left(\frac{1}{4} - \frac{1}{2}\right) + \left(-\frac{1}{4} + \frac{1}{2}\right) - 0$$

$$\frac{1}{4} + \frac{1}{4} \quad \frac{1}{2}$$