Calculus Exam Review - Semester 1

Key

Find the limit.

① $\lim_{x\to\infty} (x^2-5x-11) = \boxed{\infty}$

② $\lim_{x\to 5} \left(\frac{x+3}{x^2-15}\right) \frac{8}{10} = \frac{8}{10} = \boxed{\frac{4}{5}}$

③ $\lim_{x\to 0} \pi^2 = \boxed{\pi^2}$

④ $\lim_{x\to 3} \left(\frac{x^2-2x-3}{x-3}\right) \frac{(x-3)(x+1)}{x-3} = \boxed{4}$

⑤ $\lim_{x\to\infty} \left(\frac{10x^2+25x+1}{x^4-8}\right) = \boxed{0}$

⑥ $\lim_{x\to\infty} \left(\frac{x^4-8}{10x^2+25x+1}\right) = \boxed{\infty}$

⑦ $\lim_{x\to\infty} \left(\frac{x^4-8}{10x^4+25x+1}\right) = \boxed{\frac{1}{10}}$

⑧ $\lim_{x\to\infty} \left(\frac{\sqrt{5x^4+2x}}{x^2}\right) = \boxed{\sqrt{5}}$

$= \sqrt{\frac{5x^4+2x}{x^4}}$

⑨ $\lim_{x\to 6^+} \left(\frac{x+2}{x^2-4x-12}\right) = \infty$

$\frac{x+2}{(x-6)(x+2)}$

⑩ $\lim_{x\to 6^-} \left(\frac{x+2}{x^2-4x-12}\right) = -\infty$

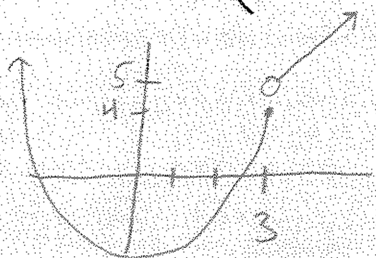
⑪ $\lim_{x\to 6} \left(\frac{x+2}{x^2-4x-12}\right) =$ DNE

⑫ $\lim_{x\to 0^+} \left(\frac{x}{|x|}\right) = 1$

⑬ $\lim_{x\to 0^-} \left(\frac{x}{|x|}\right) = -1$

⑭ $\lim_{x\to 0} \left(\frac{x}{|x|}\right) =$ DNE

(15) Let $f(x) = \begin{cases} x^2 - 5, & x \le 3 \\ x + 2, & x > 3 \end{cases}$



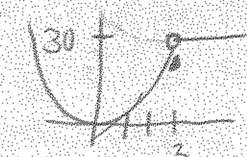
a) $\lim_{x \to 3^-} f(x) = \underline{4}$

b) $\lim_{x \to 3^+} f(x) = \underline{5}$

c) $\lim_{x \to 3} f(x) = \underline{DNE}$

(16) Is the function $f(x) = \begin{cases} 4x^2 - 2x, & x < 3 \\ 10x - 1, & x = 3 \\ 30, & x > 3 \end{cases}$ continuous @ $x = 3$?

$f(3) =$ 30
29
30



$\boxed{NO}$

(17) For what value(s) of k is the function $f(x) = \begin{cases} 3x^2 - 11x - 4 & x \le 4 \\ kx^2 - 2x - 1 & x > 4 \end{cases}$ continuous at $x = 4$?

16
3
48

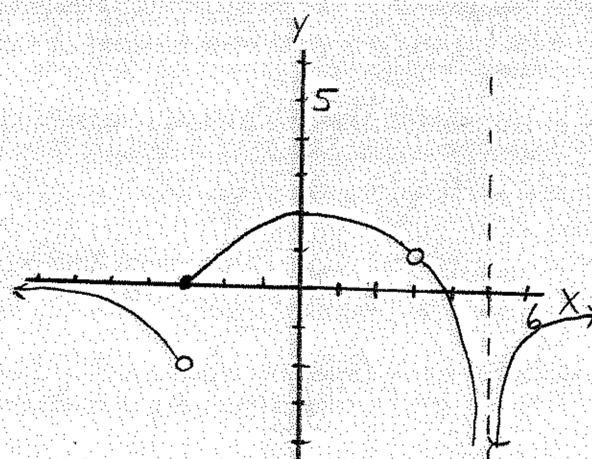
$3(4)^2 - 44 - 4 = 0$

$k(4^2) - 2(4) - 1 = 0$

$16k - 8 - 1 = 0 \rightarrow 16k = 9$

$\boxed{k = \frac{9}{16}}$

(18) Use the graph of $f(x)$ to find,



a) $\lim_{x \to -\infty} f(x) = \underline{0}$

b) $\lim_{x \to -3} f(x) = \underline{DNE}$

c) $\lim_{x \to 3^-} f(x) = \underline{1}$

d) $\lim_{x \to 3^+} f(x) = \underline{1}$

e) $f(3) = \underline{DNE}$

f) Any discontinuities

@ $x = -3$ $x = 3$ $x = 5$

Find the derivative of each equation.

(19) $y = 11x^7$

$\frac{dy}{dx} = \boxed{77x^6}$

(20) $f(x) = \frac{1}{2}(x^{12} + 17)$

$\boxed{f'(x) = 6x^{11}}$

(21) $y = \sqrt{x} + \frac{1}{x^3}$

$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{3}{x^4}}$

(22) $y = 6x^{-7} - 4\sqrt{x}$

$$\boxed{\frac{dy}{dx} = \frac{-42}{x^8} - \frac{2}{\sqrt{x}}}$$

(23) $y = e^{10} + \pi^3 - 7$

$$\boxed{\frac{dy}{dx} = 0}$$

(24) $f(x) = (4x^2+1)^2$

$$f'(x) = 2(4x^2+1)(8x)$$

$$\boxed{f'(x) = 16x(4x^2+1)}$$

(25) $y = (x+1)^3$

$$\boxed{\frac{dy}{dx} = 3(x+1)^2}$$

(26) $y = 18x^3 + 12x + 11$

$$\boxed{\frac{dy}{dx} = 54x^2 + 12}$$

(27) $g(x) = \sqrt{x} + \sqrt[3]{x} - \sqrt[3]{x^2}$

$$g(x) = x^{\frac{1}{2}} + x^{1/3} - x^{2/3}$$

$$g'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-1/3}$$

$$\boxed{g'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x}}}$$

(28) $f(x) = (6x^2+3)(12x-4)$

$$f'(x) = 12x(12x-4) + (6x^2+3)(12)$$

$$= 144x^2 - 48x + 72x^2 + 36$$

$$\boxed{f'(x) = 216x^2 - 48x + 36}$$

(29) $y = (3 - x - 2x^3)(6 + x^4)$

$$\frac{dy}{dx} = (-1 - 6x^2)(6 + x^4) + (3 - x - 2x^3)(4x^3)$$

$$= -6 - 36x^2 - x^4 - 6x^6 + 12x^3 - 4x^4 - 8x^6$$

$$\boxed{\frac{dy}{dx} = -6 - 36x^2 + 12x^3 - 5x^4 - 14x^6}$$

(30) $h(x) = (x^2 + 8x - 4)(2x^{-2} + x^{-4})$

$$h'(x) = (2x+8)(2x^{-2} + x^{-4}) + (x^2 + 8x - 4)(-4x^{-3} - 4x^{-5})$$

$$= \cancel{4x^{-1}} + \cancel{16x^{-2}} + \cancel{2x^{-3}} + \cancel{8x^{-4}} - \cancel{4x^{-1}} - \cancel{32x^{-2}} + \cancel{16x^{-3}} - \cancel{4x^{-3}} - \cancel{32x^{-4}} + 16x^{-5}$$

$$= -16x^{-2} + 14x^{-3} - 24x^{-4} + 16x^{-5}$$

$$\boxed{h'(x) = \frac{-16}{x^2} + \frac{14}{x^3} - \frac{24}{x^4} + \frac{16}{x^5}}$$

(31) $f(x) = (3x^2+1)^{10}$

$f'(x) = 10(3x^2+1)^9(6x)$

$\boxed{f'(x) = 60x(3x^2+1)^9}$

(32) $f(x) = \left(\frac{x}{x+1}\right)^4$

$f'(x) = 4\left(\frac{x}{x+1}\right)^3\left(\frac{1(x+1) - x(1)}{(x+1)^2}\right)$

$\boxed{f'(x) = \left(\frac{4}{(x+1)^2}\right)\left(\frac{x}{x+1}\right)^3}$

(33) $f(x) = 8\sqrt{x^4-4x^2}$

$= 4(x^4-4x^2)^{-\frac{1}{2}}(4x^3-8x)$

$\boxed{f'(x) = \frac{16x^3-32x}{\sqrt{x^4-4x^2}}}$

(34) $y = (x^2+x)^{100}$

$\frac{dy}{dx} = 100(x^2+x)^{99}(2x+1)$

$\boxed{\frac{dy}{dx} = (200x+100)(x^2+x)^{99}}$

(35) $f(x) = \sqrt{\left(\frac{x^2+1}{x^2-1}\right)}$

$f'(x) = \frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{-1/2}\left(\frac{2x(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}\right)$

$= \left(\frac{1}{2}\sqrt{\frac{x^2-1}{x^2+1}}\right)\left(\frac{2x^3-2x-2x^3-2x}{(x^2-1)^2}\right)$

$\boxed{= \left(\frac{1}{2}\sqrt{\frac{x^2-1}{x^2+1}}\right)\left(\frac{-4x}{(x^2-1)^2}\right)}$

(36) $y = \ln(x^4+8)$

$\boxed{\frac{dy}{dx} = \frac{4x^3}{x^4+8}}$

(37) $f(x) = e^{3x} - 3^{ex}$

$\boxed{f'(x) = 3e^{3x} - (\ln 3)(3^{ex})e}$

(38) $f(x) = x^5 \cdot 5^x$

$f'(x) = 5x^4 \cdot 5^x + x^5(\ln 5)5^x$

$\boxed{= (x^4)(5^x)(5 + x\ln 5)}$

(39) $y = \log_{12}(x^3)$

$\frac{dy}{dx} = \frac{3x^2}{(\ln 12)(x^3)}$

$\boxed{\frac{dy}{dx} = \frac{3}{x\ln 12}}$

(40) $f(x) = e^{\pi x} - \ln e^{\pi x}$

$f'(x) = e^{\pi x}\cdot\pi - \pi$

$\boxed{= \pi(e^{\pi x} - 1)}$