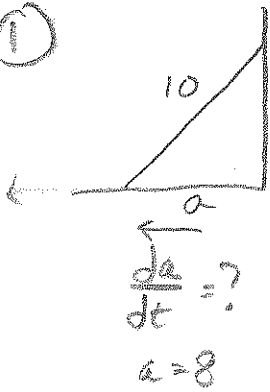


# Calculus - Related Rates - w.s. #2 Answers w/work

①



$$b \downarrow \frac{db}{dt} = -2 \frac{\text{m}}{\text{s}}$$

$$b = 6$$

$$a^2 + b^2 = 10^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 0$$

$$2(8) \frac{da}{dt} + 2(6)(-2) = 0$$

$$16 \frac{da}{dt} - 24 = 0$$

$$\frac{da}{dt} = \frac{24}{16} = \boxed{\frac{3}{2} \text{ m/s}}$$

②  $V_{\text{cone}} = \frac{1}{3} \pi r^2 h$

$$h = 3r$$

$$V = \frac{1}{3} \pi r^2 (3r)$$

$$r = 3''$$

$$\frac{dV}{dt} = 9\pi \frac{\text{in}^3}{\text{min}}$$



$$V = \pi r^3$$

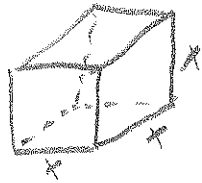
$$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow 9\pi = 3\pi (3^2) \frac{dr}{dt}$$

$$\boxed{\frac{1}{3} \frac{\text{in}}{\text{min}} = \frac{dr}{dt}}$$

③  $V = x^3$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$



$$\frac{dV}{dt} = 300 \frac{\text{in}^3}{\text{min}}$$

$$\frac{dx}{dt} = ?$$

$$x = 10$$

$$300 = 3(10^2) \frac{dx}{dt}$$

$$\boxed{1 = \frac{dx}{dt}}$$

$$S.A. = 6x^2$$

$$\frac{dSA}{dt} = 12x \frac{dx}{dt}$$

$$\frac{dSA}{dt} = 12(5)(4) = \boxed{240 \frac{\text{in}^2}{\text{min}}}$$

$$6x^2 = 150$$

$$x^2 = 25$$

$$x = 5$$

$$300 = 3(5^2) \frac{dx}{dt}$$

$$4 = \frac{dx}{dt} \text{ when } x = 5$$

$$④ \frac{dy}{dt} = ? \quad x=4 \quad \frac{dx}{dt} = 3$$

$$a) \frac{dy}{dt} = \frac{(2x \frac{dx}{dt} - \frac{dx}{dt})(x-3) - (x^2 - x - 2) \frac{dx}{dt}}{(x-3)^2} = \frac{(8 \cdot 3 - 3)(1) - (16 - 4 - 2)(3)}{(4-3)^2} = 21 - 30 = \boxed{-9 = \frac{dy}{dt}}$$

$$4x^2 + \frac{1}{2}x^{-1}$$

$$b) \frac{dy}{dt} = 8x \frac{dx}{dt} - \frac{1}{2}x^{-2} \frac{dx}{dt} = 32 \cdot 3 - \frac{1}{2} \cdot \frac{1}{16} \cdot 3 = \boxed{96 - \frac{3}{32} \approx 95.91}$$

$$c) \frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 6x \frac{dx}{dt} = 3 \cdot 16 \cdot 3 - 6 \cdot 4 \cdot 3 = \boxed{72}$$

$$d) y = 2x + x^{-2}$$

$$\frac{dy}{dt} = 2 \frac{dx}{dt} - 2x^{-3} \frac{dx}{dt} = 2 \cdot 3 - 2 \cdot \frac{1}{4^3} \cdot 3 = \boxed{6 - \frac{6}{64} \approx 5.91}$$

$$⑤ \begin{aligned} d &= 2h \\ 2r &= 2h \\ r &= h \end{aligned}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi h^3$$

$$h = 10$$

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt} = \pi (100) \frac{1}{4} = \boxed{25\pi \frac{m^3}{min} \approx 78.54 \frac{m^3}{min}}$$

$$\frac{dh}{dt} = \frac{1}{4} \frac{m}{min}$$

$$\frac{dV}{dt} = ?$$

⑥



$$\frac{dV}{dt} = 4\pi \frac{\text{cm}^3}{\text{sec}}$$

$$\frac{dr}{dt} = 2 \frac{\text{cm}}{\text{sec}}$$

$$V = \pi r^2 h$$

$$\frac{dh}{dt} = ?$$

$$V = 36\pi \text{ cm}^3$$

$$r = 3 \text{ cm}$$

$$36\pi = \pi \cdot 9 \cdot h$$

$$4 = h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$4\pi = 2\pi(3)(2)(4) + \pi \cdot 9 \frac{dh}{dt}$$

$$4\pi = 48\pi + 9\pi \frac{dh}{dt}$$

$$\frac{-44\pi}{9\pi} = \frac{dh}{dt}$$

$$\frac{-44}{9} = \frac{dh}{dt} \text{ decreasing}$$

⑦

a)  $P = 2L + 2w$

$$\frac{dP}{dt} = 2 \frac{dL}{dt} + 2 \frac{dw}{dt}$$

$$\frac{dP}{dt} = 2(2) + 2(-4)$$

$$= 4 - 8 = \boxed{-4 = \frac{dP}{dt}}$$



$$\frac{dL}{dt} = 2 \quad \frac{dw}{dt} = -?$$

$$L = 5 \quad w = 10$$

$$A = Lw = 50 \quad L = \frac{50}{w}$$

$$\frac{dA}{dt} = \frac{dL}{dt} w + \frac{dw}{dt} L$$

$$0 = 2(10) + \frac{dw}{dt}(5)$$

$$-20 = 5 \frac{dw}{dt}$$

$$-4 = \frac{dw}{dt}$$

b)  $0 = 2(2) + 2 \frac{dw}{dt}$

$$-4 = 2 \frac{dw}{dt}$$

$$-2 = \frac{dw}{dt}$$

$$\frac{dA}{dt} = \frac{dL}{dt} w + \frac{dw}{dt} L$$

$$0 = 2\left(\frac{50}{L}\right) + -2(L)$$

$$\left(0 = \frac{100}{L} - 2L\right) \cdot L$$

$$= 100 - 2L^2$$

$$50 = L^2$$

$$\sqrt{50} = L$$

$$5\sqrt{2} = L$$

$$\frac{50}{5\sqrt{2}} = \frac{5\sqrt{2} \cdot w}{5\sqrt{2}}$$

$$\frac{10}{\sqrt{2}} = w = 5\sqrt{2}$$

$$\boxed{L \times w = 5\sqrt{2} \times 5\sqrt{2}}$$