CALC. Final Review ANSWERS

(4) $\frac{8}{3}x^{-2}$

$\frac{-16}{3}x^{-3}$

$\frac{-16}{3x^3}$

(5) $\frac{-9}{27}x^{-6}$

$\frac{-1}{3}x^{-6}$

$2x^{-7}$

$\frac{2}{x^7}$

(6) $6x^{\frac{1}{2}}$

$3x^{-\frac{1}{2}}$

$\frac{3}{\sqrt{x}}$

(7) $\frac{4}{3}x^2 - \frac{5}{3}x + 2$

$\cancel{\frac{8}{3}x - \frac{5}{3}}$

$\frac{8x-5}{3}$

(8) $\frac{1}{2}x - 3 + x^{-1}$

$\frac{1}{2} - \frac{1}{x^2}$

(9) $\frac{2x^3+6x^2-8}{(x+2)^2}$

(10) $4x^3 - \frac{9}{2}x^2 + 10x - 6$

(11) $x - 3 + 10x^{-1} - 5x^{-2}$

$1 - \frac{10}{x^2} + \frac{10}{x^3}$

(12) $2x^3 + 8x^2 - x^2 - 4x$

$2x^3 + 7x^2 - 4x$

$6x^2 + 14x - 4$

(16) $x^5 - \cancel{4x^4} - \cancel{6x^3} - \cancel{5x^4} + \cancel{20x^3} + \cancel{30x^2} - \cancel{3x^3} + \cancel{12x^2} + 18x$

$x^5 - 9x^4 + 11x^3 + 42x^2 + 18x$

$5x^4 - 36x^3 + 33x^2 + 84x + 18$

$\frac{x^2}{2x}$

$\frac{1}{2}x$

(17) $\frac{3(2x+3) - 2(3x-2)}{(2x+3)^2} = \frac{6x+9-6x+4}{(2x+3)^2} = \frac{13}{(2x+3)^2}$

(18) $\frac{(2x-4)(x^2-1) - (x^2-4x-2)(2x)}{(x^2-1)^2} = \frac{\cancel{2x^3} - \cancel{4x^2} - 2x + 4 - \cancel{2x^3} + \cancel{8x^2} + 4x}{(x^2-1)^2} = \frac{4x^2+2x+4}{(x^2-1)^2}$

(19) $\sqrt{x} - x^{-\frac{1}{2}}$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$

(20) $x^{\frac{5}{3}} - x^{\frac{2}{3}} + x^{-\frac{1}{3}}$

$\frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{3}} - \frac{1}{3}x^{-\frac{4}{3}}$

(21) $\frac{3x^2 - 9x - 2x + 6}{} = \frac{3x^2 - 11x + 6}{x+4}$

$\frac{(6x-11)(x+4) - 3x^2 + 11x - 6}{(x+4)^2}$

$\frac{6x^2 - 11x + 24x - 44 - 3x^2 + 11x - 6}{(x+4)^2}$

$\frac{3x^2 + 24x - 50}{(x+4)^2}$

(37) $\frac{1}{2}x^2 - \frac{3}{2}x - 2 - \frac{1}{2}x^{-1}$

$f'(x) = x - \frac{3}{2} + \frac{1}{2}x^{-2}$

$f''(x) = 1 - x^{-3}$

$1 - \frac{1}{x^3}$

(38) $f'(x) = \frac{(x-4) - x}{(x-4)^2}$

$f'(x) = -4(x-4)^{-2}$

$f''(x) = 8(x-4)^{-3}$

10 $\cancel{5x^2 \ldots}$ 10

$3x^3 \ldots 12$

Chain Rule

① $4(3x-8)^3 \cdot 3$

$12(3x-8)^3$

② $5(3x^2+2)^4 \cdot 6x$

$30x(3x^2+2)^4$

③ $40(x^2+x-1)^9(2x+1)$

$40(2x+1)(x^2+x-1)^9$

④ $-\frac{15}{2}(4-9x)^{1/2}(-9)$

$\frac{135}{2}\sqrt{4-9x}$

⑤ $y=(3x-2)^{-1}$

$-(3x-2)^{-2}$

$\boxed{\frac{-3}{(3x-2)^2}}$

⑥ $-(x^2-5x-6)^{-2}$

$2(x^2-5x-6)^{-3}(2x-5)$

$\frac{4x-10}{(x^2-5x-6)^3}$

⑩ $3x^2(5x-1)^4+x^3\cdot 4(5x-1)^3\cdot 5$

$3x^2(5x-1)^4+20x^3(5x-1)^3$

$x^2(5x-1)^3[3(5x-1)+20x]$ $15x-3$

$x^2(5x-1)^3(35x-3)$

⑦ $\left(\frac{2-x}{2}\right)^{-2}$

$(1-\frac{1}{2}x)^{-2}$

$\cancel{-}2(1-\frac{1}{2}x)^{-3}(\cancel{-}\frac{1}{2})$

$\frac{1}{(1-\frac{1}{2}x)^3}=\frac{1}{\left(\frac{2-x}{2}\right)^3}$

⑫ $\frac{1}{3}(3x^3-4x+2)^{-2/3}(9x^2-4)$

Implicit Differentiation

1) $xy = 4 \quad (-4,-1)$

$1 \cdot y + y' \cdot x = 0$

$y' = \frac{-y}{x}$ $\quad \boxed{\frac{-1}{4}}$

2) $x^2 - y^3 = 0 \quad (1,1)$

$2x - 3y^2 y' = 0$

$y' = \frac{-2x}{-3y^2}$ $\quad \boxed{\frac{2}{3}}$

3) $x^{1/2} + y^{1/2} = 9 \quad (16, 25)$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y' = 0$ $\quad \boxed{\frac{-5}{4}}$

$y' = \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}} = \frac{-\sqrt{y}}{\sqrt{x}}$

4) $x^3 - xy + y^2 = 4 \quad (0,-2)$

$3x^2 - (y + y' \cdot x) + 2y \cdot y' = 0$

$y'(\cancel{-x+2y}) = \frac{-3x^2 + y}{-x+2y}$

$y' = \frac{-2}{-4} = \boxed{\frac{1}{2}}$

9) $x^3 y - y = x$

$3x^2 y + y' \cdot x^3 - y' = 1$

$y'(\cancel{x^3 - 1}) = \boxed{\frac{1 - 3x^2 y}{x^3 - 1}}$

13) $1 - xy = x - y$

$+y' \cdot x$

$-(y + y' \cdot x) = 1 - y'$

$\boxed{\frac{-y-1}{x-1}} = y'(\cancel{x-1})$

RELATED RATES

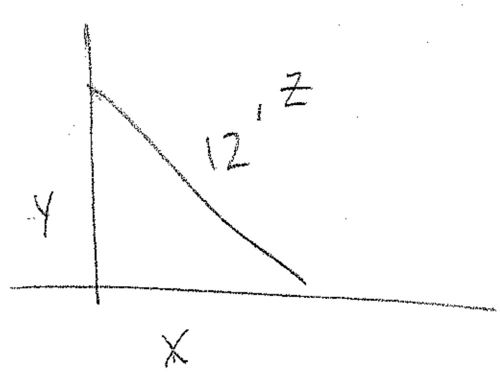
3) a) $V = \left(\frac{1}{3}\pi r^2\right)(h)$ $\frac{r}{h} = \frac{6}{10} = \frac{3}{5}$ $r = \frac{3h}{5}$

$$\frac{dV}{dt} = \left(\frac{2}{3}\pi r \cdot \frac{dr}{dt}\right)h + \frac{dh}{dt}\left(\frac{1}{3}\pi r^2\right)$$

a) $$\frac{dV}{dt} = \frac{2}{3}\pi(6)\cdot(6)\cdot 10 + \left(\overset{-1}{\cancel{-12}}\right)\left(\frac{1}{3}\pi(6)^2\right)$$

b) $$\frac{dV}{dt} = \frac{2}{3}\pi(6)\cdot(-3)\cdot 10 + \left(\overset{1}{\cancel{12}}\right)\left(\frac{1}{3}\pi(6)^2\right)$$

6)



a) $\frac{dy}{dt} = ?$ $\frac{dx}{dt} = 2$

$z = 12,\ x = \sqrt{108},\ y = 6$

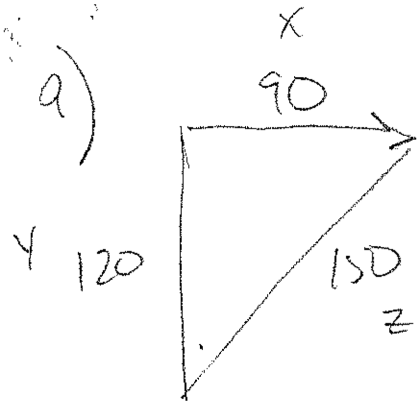
$x^2 + y^2 = z^2$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2\sqrt{108}(2) + 2(6)\left(\frac{dy}{dt}\right) = 2(12)\cdot 0$$

$$\frac{dy}{dt} = \frac{-4\sqrt{108}}{12} = \boxed{\frac{-\sqrt{108}}{3}\ \frac{ft}{sec}}$$

9)



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(90)(30) + 0 = 2(150)\frac{dz}{dt}$$

$$\boxed{\frac{dz}{dt} = 18 \text{ ft/min}}$$

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(142) $f(x) = x^3 - 3x^2 - 4x - 2$

$f'(x) = 3x^2 - 6x - 4 \rightarrow 0 = 3x^2 - 6x - 4$

??? $0 = (3x \quad)(x \quad)$

$x \approx -.53,\ 2.53$

+	−	+
INC.	DEC.	INC

$-.53$ MAX, 2.53 MIN

$f''(x) = 6x - 6 \rightarrow 0 = 6x - 6$

$x = 1$

−	+
C.DOWN	C.UP

1 INFL. pt.

INC. $(-\infty, -.53)$
$(2.53, \infty)$

DEC. $(-.53, 2.53)$

C.Up $(1, \infty)$

C.DOWN $(-\infty, 1)$

LOCAL MAX
$(-.53, -.8716)$

LOCAL MIN
$(2.53, -15.13)$

INFL. Pt. $(1, -8)$

Ind. Integrals

4) $\frac{1}{8}x^8 + C$

5) $\frac{1}{5}x^5 + \frac{1}{4}x^4 - \frac{1}{3}x^3 + C$

6) $\frac{3}{4}x^4 - \frac{4}{3}x^3 + C$

9) $2x + \frac{1}{4}x^{-4} - \frac{7}{2}x^{-2} + C$

$2x + \frac{1}{4x^4} - \frac{7}{2x^2} + C$

10) $\frac{10}{3}x^{3/2} + C$

11) $\frac{25}{6}x^{6/5} + C$

12) $\frac{4}{7}x^{7/4} + 4x^{1/4} + C$

U Subst.

1) $\frac{2}{3}(x-2)^{3/2} + C$

2) $\frac{1}{24}(2x+3)^{12} + C$

3) $\frac{2}{15}(5x-1)^{3/2} + C$

4) $\frac{1}{8}(6x+1)^{3/2} + C$

6) $\frac{-1}{16}(8x-1)^{-2} + C$

7) $\frac{1}{14}(x^2+6)^7 + C$

8) $\frac{4}{9}(3x^3-1)^{3/2} + C$

U-Substitution cont.

(12) $u = x^2+10x+4$

$du = 2x+10\,dx$ $\quad \int \frac{3}{2} u^{\frac{1}{2}}\,du$

$\frac{3}{2}du = 3x+15\,dx$ $\quad u^{3/2}+C \rightarrow (x^2+10x+4)^{3/2} + C$

Fundamental Theorem of Calc.

(1) $\frac{3}{2}x^2\Big|_0^1 = \frac{3}{2} - 0 = \frac{3}{2}$

(2) $\frac{1}{2}x^2 - 5x\Big|_{-2}^{3} = \left(\frac{9}{2} - 15\right) - (2+10) = \frac{-21}{2} - \frac{24}{2} = \frac{-45}{2}$

$\frac{30}{2}$

(3) $\frac{1}{3}x^3 + x^2 - x\Big|_{-1}^{4} = \left(\frac{64}{3}+16-4\right) - \left(\frac{-1}{3}+1+1\right) = \frac{65}{3} + \frac{30}{3} = \frac{95}{3}$

(4) $u = 2x-5$ $\quad \int \frac{1}{2}u^2\,du$

$du = 2\,dx$ $\quad \frac{1}{6}u^3 \rightarrow \frac{1}{6}(2x-5)^3\Big|_0^2 = \frac{-1}{6} + \frac{125}{6} = \frac{124}{6}$

$\frac{1}{2}du = dx$

(5) $\int 4x^{-2}+1\,dx \rightarrow -4x^{-1}+x\Big|_2^3 \rightarrow \frac{-4}{x}+x\Big|_2^3 = \left(\frac{-4}{3}+3\right) - (-2+2)$

$\frac{-4}{3}+\frac{9}{3} - 0 = \frac{5}{3}$

(6) $\int_{-2}^{-1} x - x^{-2}\,dx = \frac{1}{2}x^2 + \frac{1}{x}\Big|_{-2}^{-1} = \left(\frac{1}{2}-1\right) - \left(2-\frac{1}{2}\right) = \frac{-1}{2} - \frac{3}{2} = -2$

Area of Region Between Two Curves

(1) $x^2+2x+1 = 2x+5$ $\quad \int_{-2}^{2} (2x+5) - (x^2+2x+1)\,dx$

$x^2-4=0$ $\quad = \int -x^2+4\,dx$

$x=\pm 2$ $\quad -\frac{1}{3}x^3+4x\Big|_{-2}^{2} = \left(\frac{-8}{3}+8\right) - \left(\frac{8}{3}-8\right) = \frac{-16}{3}+16$

$\frac{-16}{3}+\frac{48}{3} = \frac{32}{3}$

(2) $x^2-4x+3=-x^2+2x+3$

$2x^2-6x=0$

$2x(x-3)=0$

$x=0 \quad x=3$

$\int_0^3 (-x^2+2x+3)-(x^2-4x+3)\,dx$

$\int_0^3 -2x^2+6x\,dx$

$-\frac{2}{3}x^3+3x^2\Big|_0^3 = \left(\frac{-54}{3}+27\right)-0$

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(3) $x^2=x^3$

$0=x^3-x^2$

$0=x^2(x-1)$

$x=0 \quad x=1$

$\int_0^1 x^2-x^3\,dx = \frac{1}{3}x^3-\frac{1}{4}x^4\Big|_0^1 = \frac{1}{3}-\frac{1}{4}-0$

$\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$

(4) $(x-1)^3=x-1$

$x=1,\ 2$

$\int_1^2 (x-1)-(x-1)^3\,dx + \int_0^1 (x-1)^3-(x-1)\,dx$

$\int u-u^3\,du \qquad u=x-1 \qquad du=dx$

$\frac{1}{2}u^2-\frac{1}{4}u^4 \rightarrow \frac{1}{2}(x-1)^2-\frac{1}{4}(x-1)^4\Big|_1^2$

$=\left(\frac{1}{2}-\frac{1}{4}\right)=\cancel{\frac{1}{4}}$

$\frac{1}{2}$

(5) $\int_1^5 x^{-2}\,dx = \frac{-1}{x}\Big|_1^5 = -\frac{1}{5}+1 = \frac{4}{5}$

(6) $\sqrt{3x}+1=x$

$\sqrt{3x}=x-1$

$3x=x^2-2x+1$

$0=x^2-5x+1$

$x=\frac{5\pm\sqrt{25-4(1)(1)}}{2}$

$x=\frac{5\pm\sqrt{21}}{2}$

$\int_0^{\frac{5+\sqrt{21}}{2}} (3x)^{\frac{1}{2}}+1-x\,dx$

$-6x^{-1/2}+x-\frac{1}{2}x^2\Big|_0^{\frac{5+\sqrt{21}}{2}} = 5.423$

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Volume by Disks/Washers

① $\int_0^4 \pi(4-x)^2\,dx = \int_0^4 \pi(16-8x+x^2)\,dx = 16\pi x - 4\pi x^2 + \frac{\pi}{3}x^3 \Big|_0^4$

$= \left(64\pi - 64\pi + \frac{64\pi}{3}\right) - 0 = \frac{64\pi}{3}$

$(x^2+x)(x^2+x)$
$x^4+x^3+x^3+x^2$

② $\int_0^2 \pi(x^2+x)^2\,dx = \int_0^2 \pi(x^4+2x^3+x^2)\,dx = \frac{\pi}{5}x^5 + \frac{\pi}{2}x^4 + \frac{\pi}{3}x^3 \Big|_0^2$

$= \frac{32\pi}{5} + \frac{16\pi}{2} + \frac{8\pi}{3} = \frac{192\pi}{30} + \frac{240\pi}{30} + \frac{80\pi}{30} = \frac{512\pi}{30}$

④ $\int_0^2 \pi\left((2x)^2 - (x^2)^2\right)dx = \int_0^2 \pi(4x^2 - x^4)\,dx = \frac{4\pi}{3}x^3 - \frac{\pi}{5}x^5 \Big|_0^2$

$\frac{32\pi}{3} - \frac{32\pi}{5} = \frac{160\pi}{15} - \frac{96\pi}{15} = \frac{64\pi}{15}$

Differentiation of Trig. Functions

① $y' = 3\cos(3x)$ ② $y' = \sin x + x\cos x$ ③ $y' = \sin\left(\frac{\pi}{2} - x\right)$

④ $y' = \left(\frac{x\cos x - \sin x}{x^2}\right)$ ⑤ $y' = \frac{\sin x - x\cos x}{\sin^2 x}$ ⑥ $y' = 3x^2\sin^2 x + \left(x^3(2\sin x)(\cos x)\right)$

⑦ $y' = -2\cos(2x) - 3\cos(3x)$ ⑧ $y' = 4\cos^3(x^4) \cdot -4x^3(\sin(x^4))$ ⑨ $y' = 2\sin x \cdot \cos x + 2\cos x(\ldots$

$y' = 0$