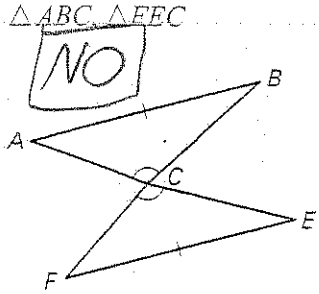
Adv. Geometry 4.3-4.5

Name Key

Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

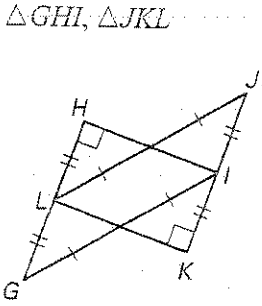
1. $\triangle ABC$, $\triangle EFC$

NO



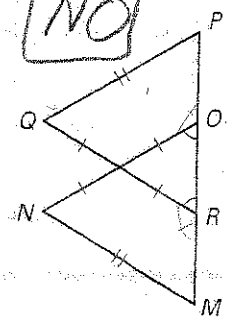
2. $\triangle GHI$, $\triangle JKL$

Yes, HL-Theorem



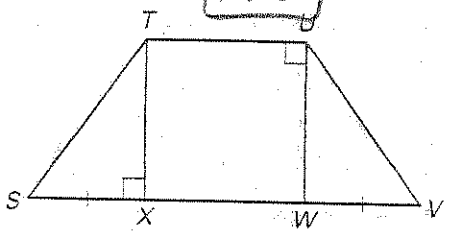
3. $\triangle MNO$, $\triangle PQR$

NO



4. $\triangle STX$, $\triangle VUW$

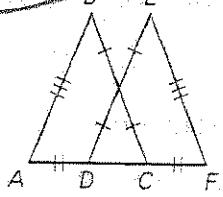
NO



Decide whether the congruence statement is true. *Explain* your reasoning.

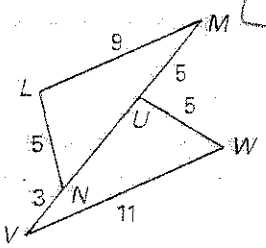
1. $\triangle ABC \cong \triangle EFD$

Yes, SSS



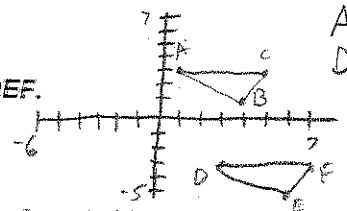
2. $\triangle LMN \cong \triangle UVW$

NO, corresponding sides not ≅



Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.

4. $A(1, 3), B(4, 1), C(5, 3), D(3, -3), E(6, -5), F(7, -3)$



$AC = 4$ $BC = \sqrt{5}$ $AB = \sqrt{13}$

$DF = 4$ $EF = \sqrt{5}$ $DE = \sqrt{13}$

$\triangle ABC \cong \triangle DEF$ by SSS

State the third congruence that must be given to prove that $\triangle ABC \cong \triangle FED$ using the indicated postulate or theorem.

5. GIVEN: $\overline{BC} \cong \overline{ED}$, $\overline{AC} \cong \overline{FD}$, ? ≅ ?

$\angle BCA \cong \angle EDF$

Use the SAS Congruence Postulate.

6. GIVEN: $\overline{AB} \cong \overline{FE}$, $\overline{AC} \cong \overline{FD}$, ? ≅ ?

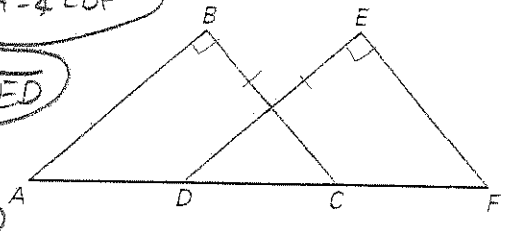
$\overline{BC} \cong \overline{ED}$

Use the SSS Congruence Postulate.

7. GIVEN: $\overline{BC} \cong \overline{ED}$, $\angle B$ is a right angle and $\angle B \cong \angle E$, ? ≅ ?

$\overline{AC} \cong \overline{FD}$

Use the HL Congruence Theorem.

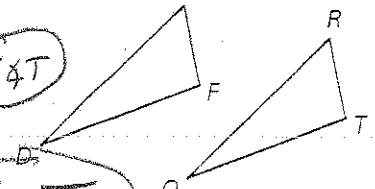


State the third congruence that is needed to prove that $\triangle DEF \cong \triangle QRT$ using the given postulate or theorem.

1. GIVEN: $\angle D \cong \angle Q, \angle F \cong \angle T, \_?\_ \cong \_?\_$ $\overline{EF} \cong \overline{RT}$ or $\overline{DE} \cong \overline{QR}$
   Use the AAS Congruence Theorem.

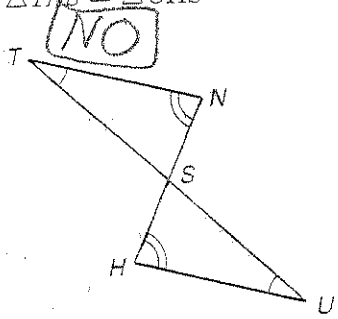
2. GIVEN: $\angle E \cong \angle R, \overline{EF} \cong \overline{RT}, \_?\_ \cong \_?\_$ $\angle F \cong \angle T$
   Use the ASA Congruence Postulate.

3. GIVEN: $\overline{DE} \cong \overline{QR}, \angle D \cong \angle Q, \_?\_ \cong \_?\_$ $\overline{DF} \cong \overline{QT}$
   Use the SAS Congruence Postulate.

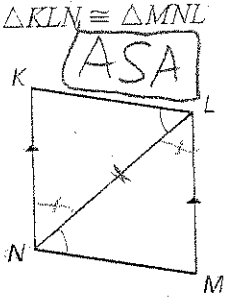


Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.

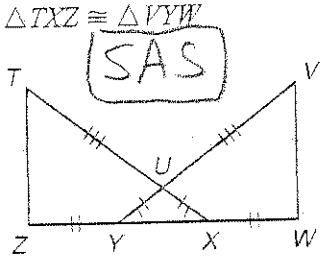
4. $\triangle TNS \cong \triangle UHS$ NO



5. $\triangle KLN \cong \triangle MNL$ ASA



6. $\triangle TXZ \cong \triangle VYW$ SAS



Tell whether you can use the given information to determine whether $\triangle JRM \cong \triangle XYZ$. *Explain* your reasoning.

7. $\overline{JM} \cong \overline{XZ}, \angle M \cong \angle Y, \angle J \cong \angle X$ NO, correspondence not right

8. $\overline{JM} \cong \overline{XZ}, \overline{JR} \cong \overline{YZ}, \angle J \cong \angle X$ NO, correspondence not correct.

9. $\angle J \cong \angle X, \angle M \cong \angle Z, \overline{RM} \cong \overline{YZ}$ AAS

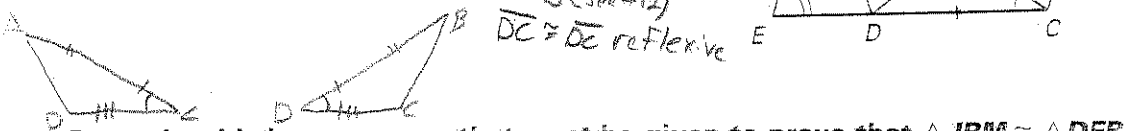
10. $\overline{JR} \cong \overline{YZ}, \overline{RM} \cong \overline{ZX}, \overline{MJ} \cong \overline{XY}$ SSS

Explain how you can prove that the indicated triangles are congruent using the given postulate or theorem.

11. $\triangle AFD \cong \triangle BFC$ by SAS $\angle AFD \cong \angle BFC$ by vertical $\angle$s

12. $\triangle ACE \cong \triangle DBA$ by AAS $DF + FB = CF + FA \rightarrow \overline{DB} \cong \overline{AC}$

13. $\triangle ACD \cong \triangle BDC$ by SAS $\angle BDC \cong \angle ACD$ ($\triangle FDC$ is isosceles) $\overline{AC} \cong \overline{DB}$ (Segs $\cong$ $\angle$s) $\overline{DC} \cong \overline{DC}$ reflexive

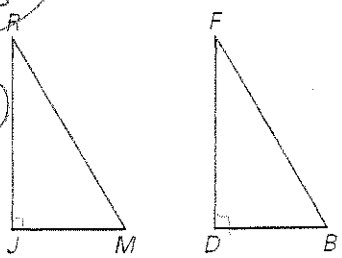


State the third congruence that must be given to prove that $\triangle JRM \cong \triangle DFB$ using the indicated postulate.

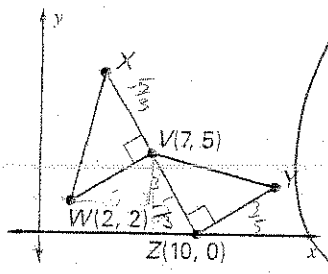
13. GIVEN: $\overline{JR} \cong \overline{DF}, \overline{JM} \cong \overline{DB}, \_?\_ \cong \_?\_$ $\overline{RM} \cong \overline{FB}$
    Use the SSS Congruence Postulate.

14. GIVEN: $\overline{JR} \cong \overline{DF}, \overline{JM} \cong \overline{DB}, \_?\_ \cong \_?\_$ $\angle J \cong \angle D$
    Use the SAS Congruence Postulate.

15. GIVEN: $\overline{RM} \cong \overline{FB}$, $\angle J$ is a right angle and $\angle J \cong \angle D, \_?\_ \cong \_?\_$
    Use the HL Congruence Theorem.
    $\overline{RJ} \cong \overline{FD}$ or $\overline{JM} \cong \overline{DB}$



6. In the figure, $\triangle VWX \cong \triangle ZYV$. Find the coordinates of X and Y.

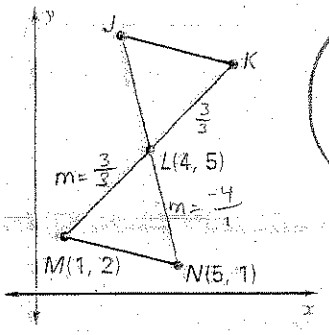


*Use ⊥ slopes

X(4, 10)

Y(15, 3)

6. In the figure, $\triangle MLN \cong \triangle KLJ$. Find the coordinates of J and K.

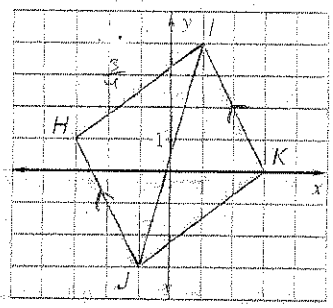


* use slopes

K(7, 8)

J(3, 9)

10. **Error Analysis** Describe and correct the error in writing a congruence statement for the triangles in the coordinate plane.



$\triangle JHI \cong \triangle JKI$

$\overline{IK} \parallel \overline{JH} \Rightarrow \angle KIJ \cong \angle HJI \Rightarrow I$ corresponds to J

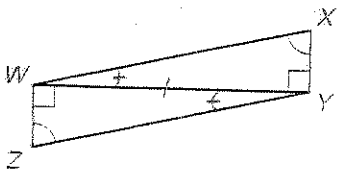
$\overline{HI} \parallel \overline{KJ} \Rightarrow \angle HIJ \cong \angle KJI \Rightarrow$

should be $\triangle HIJ \cong \triangle KJI$

15. **Proof** Complete the proof.

GIVEN: $\angle XYW \cong \angle ZWY$, $\angle WXY \cong \angle YZW$

PROVE: $\overline{WX} \parallel \overline{ZY}$

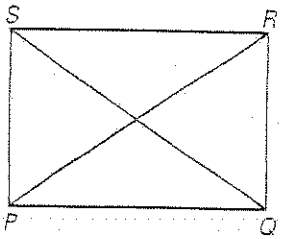


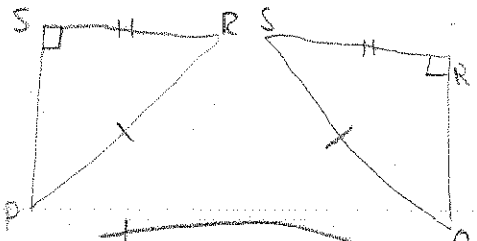
Statements	Reasons
1. $\angle XYW \cong \angle ZWY$	1. ? Given
2. $\angle WXY \cong \angle YZW$	2. ? Given
3. $\overline{WY} \cong \overline{WY}$	3. ? Reflexive
4. $\triangle XYW \cong \triangle ZWY$	4. ? AAS
5. $\angle XWY \cong \angle ZYW$	5. ? Third Angles Theorem
6. $\overline{WX} \parallel \overline{WX}$	6. ? AIA $\cong \Rightarrow$ Lines $\parallel$

9. Proof Complete the proof.

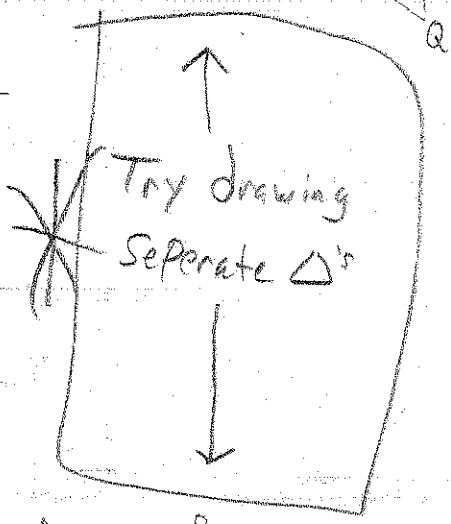
GIVEN: $\overline{QS} \cong \overline{PR}$, $\overline{PS} \perp \overline{RS}$, $\overline{QR} \perp \overline{RS}$

PROVE: $\triangle PRS \cong \triangle QSR$





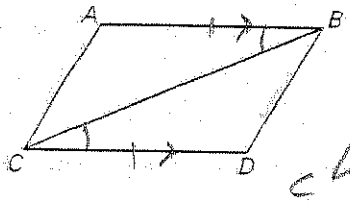
Statements	Reasons
1. $\overline{QS} \cong \overline{PR}$	1. Given
2. $\overline{PS} \perp \overline{RS}$, $\overline{QR} \perp \overline{RS}$	2. Given
3. $\angle S$ and $\angle R$ are right angles.	3. ? Def. of ⊥ Lines
4. ? $\angle R \cong \angle S$	4. Definition of a right triangle
5. $\overline{RS} \cong \overline{SR}$	5. ? Reflexive
6. $\triangle PRS \cong \triangle QSR$	6. ? HL-Theorem

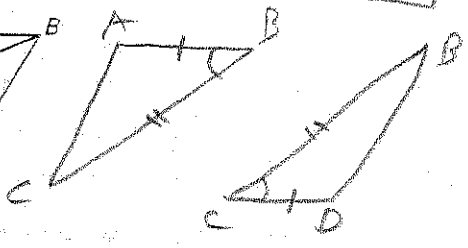


17. Proof Complete the proof.

GIVEN: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$

PROVE: $\triangle ABC \cong \triangle DCB$



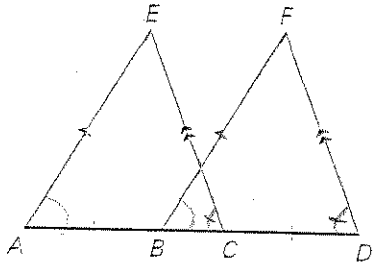


Statements	Reasons
① $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$	1. ? Given
② $\angle ABC \cong \angle DCB$	2. ? Alternate Interior angles ≅
③ $\overline{BC} \cong \overline{CB}$	3. ? Reflexive
④ $\triangle ABC \cong \triangle DCB$	4. ? SAS

5. GIVEN: $\overline{AE} \parallel \overline{BF}$, $\overline{CE} \parallel \overline{DF}$,

$\overline{AB} \cong \overline{CD}$

PROVE: $\triangle AEC \cong \triangle BFD$

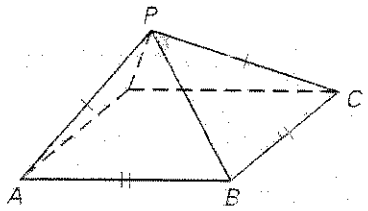


Statements	Reasons
① $\overline{AE} \parallel \overline{BF}$; $\overline{CE} \parallel \overline{DF}$	① Given
② $\overline{BC} \cong \overline{BC}$	② Reflexive
③ $AB + BC = CD + BC$	③ Addition Prop. Eq.
④ $AB + BC = AC$ $CD + BC = BD$	④ Segment Addition Post.
⑤ $\overline{AC} \cong \overline{BD}$	⑤ Substitution / Transitive
⑥ $\angle A \cong \angle FBD$ and $\angle D \cong \angle ECA$	⑥ Corresponding $\angle$'s ≅
⑦ $\triangle AEC \cong \triangle BFD$	⑦ ASA

In Exercises 1–4, refer to the diagram to write a two-column proof.

1. GIVEN: $\overline{PA} \cong \overline{PC}, \overline{AB} \cong \overline{BC}$

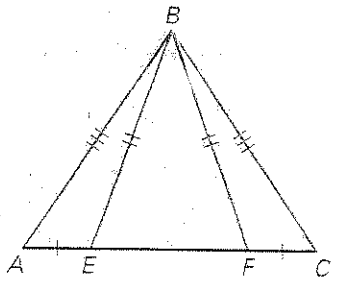
   PROVE: $\triangle PAB \cong \triangle PBC$



Statement	Reason
① $\overline{PA} \cong \overline{PC}, \overline{AB} \cong \overline{BC}$	① Given
② $\overline{PB} \cong \overline{PB}$	② Reflexive
③ $\triangle PAB \cong \triangle PBC$	③ SSS

3. GIVEN: $\overline{AE} \cong \overline{FC}, \overline{BE} \cong \overline{BF}$, $\overline{AB} \cong \overline{BC}$

   PROVE: $\triangle AFB \cong \triangle CEB$

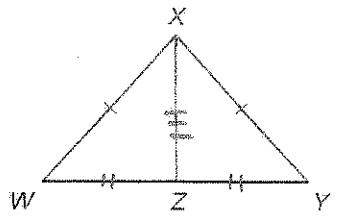


Statements	Reasons
① $\overline{AE} \cong \overline{FC}, \overline{BE} \cong \overline{BF}$, $\overline{AB} \cong \overline{BC}$	① Given
② $\overline{EF} \cong \overline{EF}$	② Reflexive
③ $\overline{AE} + \overline{EF} = \overline{FC} + \overline{EF}$	③ Addition Prop. Eq.
④ $\overline{AE} + \overline{EF} = \overline{AF}$, $\overline{FC} + \overline{EF} = \overline{CE}$	④ Segment Addition Post.
⑤ $\overline{AF} \cong \overline{CE}$	⑤ Substitution / Transitive
⑥ $\triangle AFB \cong \triangle CEB$	⑥ SSS

12. **Proof** Complete the proof.

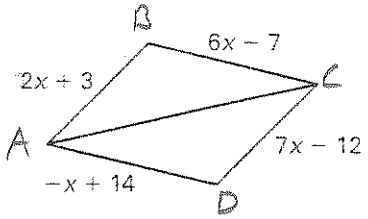
   GIVEN: $\overline{WX} \cong \overline{YX}$, Z is the midpoint of $\overline{WY}$.

   PROVE: $\triangle WXZ \cong \triangle YXZ$



Statements	Reasons
1. ? $\overline{WX} \cong \overline{YX}$; Z is midpt. of $\overline{WY}$	① Given
2. ? $\overline{WZ} \cong \overline{YZ}$	② Def. of Midpt.
3. ? $\overline{XZ} \cong \overline{XZ}$	③ Reflexive
4. ? $\triangle WXZ \cong \triangle YXZ$	④ SSS
5.	

13. Find all values of x that make the triangles congruent. *Explain.*



Case 1 → $\triangle ABC \cong \triangle ADC$

$2x+3 = -x+14$ & $6x-7 = 7x-12$

$3x = 11$ $\quad 5 = x$

$x = \frac{11}{3}$

Different x-values is not possible

so $\triangle ABC \not\cong \triangle ADC$

Case 2 → $\triangle ABC \cong \triangle CDA$

$2x+3 = 7x-12$ & $6x-7 = -x+14$

$15 = 5x$ $\quad 7x = 21$

$3 = x$ $\quad x = 3$