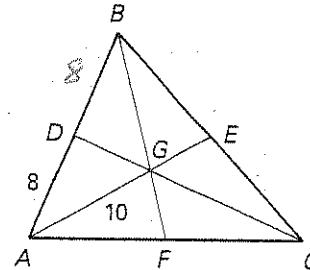


Adv. Geometry 5.4 Medians & Altitudes

Key

G is the centroid of $\triangle ABC$, $AD = 8$, $AG = 10$, and $CD = 18$. Find the length of the segment.

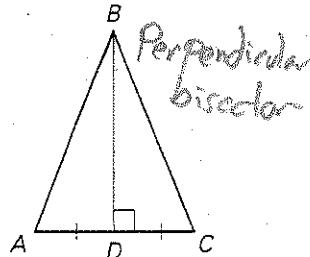
1. $\overline{BD} = 8$
2. $\overline{AB} = 16$
3. $\overline{EG} = 5$
4. $\overline{AE} = 15$
5. $\overline{CG} = 12$
6. $\overline{DG} = 6$



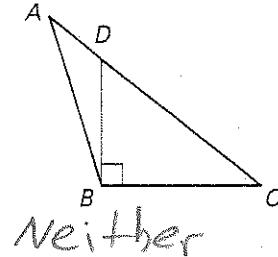
$\frac{2}{3}AE = 10$

Is \overline{BD} a perpendicular bisector of $\triangle ABC$? Is \overline{BD} a median? an altitude?

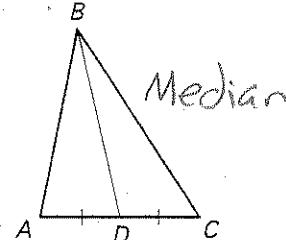
10.



11.

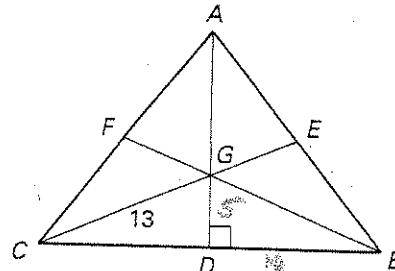


12.



G is the centroid of $\triangle ABC$, $AD = 15$, $CG = 13$, and $\overline{AD} \perp \overline{CB}$. Find the length of the segment.

1. $\overline{AG} = 10$
2. $\overline{GD} = 5$
3. $\overline{CD} = 12$
4. $\overline{GE} = 6.5$
5. $\overline{GB} = 13$



$$\begin{aligned} \frac{2}{3}CG &= 13 \\ CG &= \frac{39}{2} \\ CG &= 19.5 \end{aligned}$$

Copy and complete the statement for $\triangle LMN$ with medians \overline{LQ} , \overline{NP} , and \overline{MO} , and centroid R.

6. $MR = ? MO$
7. $RQ = ? LQ$

Point L is the centroid of $\triangle NOM$. Use the given information to find the value of x.

$$9. OL = 8x \text{ and } OQ = 9x + 6 \quad (X=3)$$

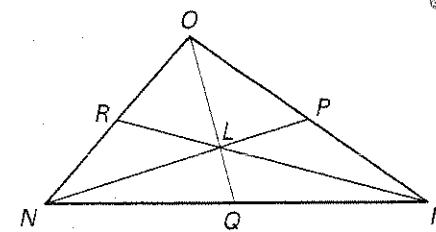
$$10. NL = x + 4 \text{ and } NP = 3x + 3 \quad (X=1)$$

$$11. ML = 10x - 4 \text{ and } MR = 12x + 18 \quad (X=6)$$

$$3(10x-4) = 12x+18 \quad (X=6)$$

$$30x-12 = 12x+18 \quad (X=6)$$

$$18 = 18x \quad (X=6)$$



$$\begin{aligned} ① \frac{2}{3}(9x+6) &= 8x \\ 6x+4 &= 8x \\ 4 &= 2x \\ 2x &= 4 \\ X &= 2 \end{aligned}$$

$$\begin{aligned} ② \frac{2}{3}(3x+3) &= x+4 \\ 2x+2 &= x+4 \\ x &= 2 \end{aligned}$$

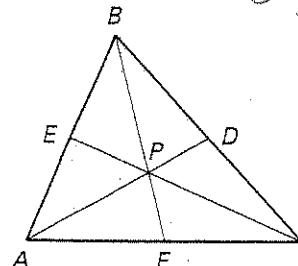
Complete the sentence with **always**, **sometimes**, or **never**.

14. The altitude from the vertex angle of an isosceles triangle is ? the median. *Always*
15. The median to any side of an equilateral triangle is ? the angle bisector. *Always*

In Exercises 1–4, point P is the centroid of $\triangle ABC$.

Use the given information to find the value(s) of x .

1. $BP = x + 4$, $BF = x^2 - 1$
2. $CP = 3x + 5$, $CE = x^2 + 2$
3. $AP = x^2 - 2$, $PD = 2x - 3$
4. $CP = x^2 + 1$, $PE = 4x - 3$



$$\textcircled{1} \frac{2}{3}(x^2 - 1) = x + 4$$

$$x^2 - 1 = \frac{3}{2}x + 6$$

$$x^2 - \frac{3}{2}x - 7 = 0$$

$$x = 3.5 \text{ or } -2$$

$$\textcircled{2} \frac{2}{3}(x^2 + 2) = 3x + 5$$

$$x^2 + 2 = \frac{9}{2}x + \frac{15}{2}$$

$$x^2 - \frac{9}{2}x - \frac{11}{2} = 0$$

$$x = 5.5 \text{ or } -1$$

In Exercises 5–8, use the following information to find the area of the triangle described.

The formula below can be used to find the area A of a triangle using the measures of the medians m .

$$A = \frac{4}{3}\sqrt{s(s - m_1)(s - m_2)(s - m_3)}, \text{ where } s = \frac{1}{2}(m_1 + m_2 + m_3).$$

$$5. m_1 = 3, m_2 = 6, m_3 = 7$$

$$S = \frac{1}{2}(3 + 6 + 7)$$

$$S = 8$$

$$A = \frac{4}{3}\sqrt{8(8-3)(8-6)(8-7)}$$

$$= \frac{4}{3}\sqrt{8 \cdot 5 \cdot 2 \cdot 1}$$

$$A = \frac{4}{3}\sqrt{80}$$

$$= \frac{4}{3}\sqrt{16 \cdot 5}$$

$$= \frac{4}{3} \cdot 4 \cdot \sqrt{5} \quad \text{or}$$

$$A = \frac{16}{3}\sqrt{5}$$

$$6. m_1 = 19, m_2 = 17, m_3 = 10$$

$$S = \frac{1}{2}(19 + 17 + 10)$$

$$= \frac{1}{2}(46) = 23 \cdot 5$$

$$A = \frac{4}{3}\sqrt{23(23-19)(23-17)(23-10)}$$

$$= \frac{4}{3}\sqrt{23(4)(6)(13)}$$

$$A = \frac{4}{3}\sqrt{7176}$$

$$\textcircled{3} AP = 2 \cdot PD$$

$$x^2 - 2 = 2(2x - 3)$$

$$x^2 - 2 = 4x - 6$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$x = 2$$

$$\textcircled{4} x^2 + 1 = 2(4x - 3)$$

$$x^2 + 1 = 8x - 6$$

$$x^2 - 8x + 7 = 0$$

$$(x-7)(x-1) = 0$$

$$x = 1 \quad x = 7$$