

Precalculus 2.4 Complex Numbers

The imaginary unit, i , is defined as $i = \sqrt{-1}$

Powers of i : $i^2 = -1$ $i^3 = -i$ $i^4 = 1$
 $i^2 \cdot i = -1 \cdot i = -i$ $i^2 \cdot i^2 = -1 \cdot -1 = 1$

-to simplify $\sqrt{-45}$, first get in terms of i

$$\sqrt{45} * \sqrt{-1} = \sqrt{9} * \sqrt{5} * i = 3i\sqrt{5}$$

$$\sqrt{9 \cdot 5}$$

$$3i\sqrt{5}$$

Example

Simplify into terms of i $\sqrt{-60}$

$$\sqrt{60} \cdot \sqrt{-1}$$

$$\sqrt{4} \cdot \sqrt{15} \cdot i$$

$$2i\sqrt{15}$$

$$\sqrt{3} \cdot \sqrt{5}$$

$$\begin{array}{r} 60 \\ \hline 1 \cdot 60 \\ 2 \cdot 30 \\ 3 \cdot 20 \\ 4 \cdot 15 \\ 5 \cdot 12 \\ 6 \cdot 10 \end{array}$$

- a complex number written in standard form is both a and b are real numbers

$$5 + 6i$$

$$3 - 2i$$

$$a + bi$$

real part

imaginary part

- two complex numbers are equal if all their parts are equal.

$$a + bi = c + di \quad \text{if and only if} \quad a = c \quad \text{and} \quad b = d$$

Find the real numbers a and b so that,

$$a + (b - 2)i = 7 - 3i$$

$$a = 7$$

$$b - 2 = -3$$

$$b = -1$$

Operations with Complex Numbers

-When **adding, subtracting, or multiplying** treat i like a variable and go with like terms.

Examples

Perform the operation and write answer in standard form

$$(7 - 6i) - (4 + 5i)$$

$$3 - 11i$$

$$(2 + 8i)(3 - i)$$

$$6 - 2i + 24i - 8i^2$$

$$6 + 22i + 8$$

$$14 + 22i$$

$a + bi$

$-8(-1)$

↑

-When **dividing** by a complex number, you multiply the *numerator and denominator* by conjugate of the denominator.

^{the} -**Complex conjugate** is obtained by switching the sign of the imaginary term. For example, the conjugate of $2+3i$ is $2-3i$

Example

Perform the operation and write answer in standard form

$$\frac{(2+3i)}{(4-5i)} \cdot \frac{(4+5i)}{(4+5i)} = \frac{8+10i+12i+15i^2}{16+20i-20i-25i^2} = \frac{-7+22i}{41}$$

$= \frac{-7}{41} + \frac{22}{41}i$

#2 $(b+3)i$

$$\frac{\sqrt{a} \cdot \sqrt{b}}{\sqrt{a \cdot b}}$$

$$a > 0$$

$$b > 0$$

(10) $\sqrt{-6} \cdot \sqrt{-2} \rightarrow i\sqrt{6} \cdot i\sqrt{2}$

$$i^2 \sqrt{12}$$

$$-1 \cdot 2\sqrt{3}$$

$$\boxed{-2\sqrt{3}}$$

~~$\sqrt{-6 \cdot -2}$~~

~~$\sqrt{12}$~~

~~$2\sqrt{3}$~~