

Precalculus Sec. 2.5

Fundamental Theorem of Algebra

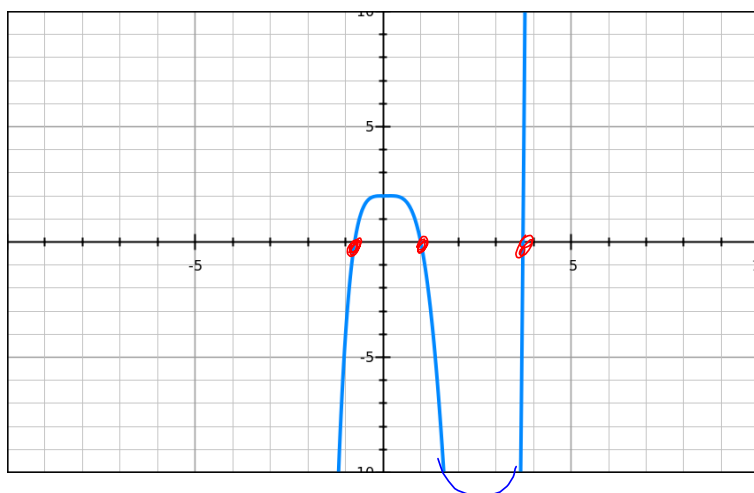
-the set of complex numbers includes all the Real numbers, rational, irrational and integers

Fundamental Theorem of Algebra If $f(x)$ is a polynomial of degree n , where $n > 0$, then f has at least one zero in the complex number system.

degree \uparrow

integer

-this means that a degree n polynomial has exactly n zeros. These zeros can be real or complex and they may be repeated (multiplicity).



This is the graph of

$$f(x) = x^5 - 4x^4 + x^3 + 2$$

What is the degree of the function? 5

How many zeros does the function have? 5

According to the graph, how many real zeros does the function have? 3

How many complex zeros does the function have? 2

How many zeros does $f(x) = x^2 - 6x + 9$ have?
Find them:

How many zeros does $x^3 + 4x$ have?
Find them:

Rational Zeros Test- the possible real zeros of a polynomial with integer coefficients is

$$\frac{\pm \text{factors of constant}}{\pm \text{factors of leading coefficient}}$$

How many zeros does $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ have?

Find possible zeros:

Find the zeros:

1 mult. 2, -2

$$(x+2)(x^4 - 2x^3 + 3x^2 - 8x + 4)$$

$$(x+2)(x-1)(x^3 - x^2 + 4x - 4) = 0$$

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 1 & 2 & -12 & 8 \\ & & -2 & 4 & -10 & 16 & -8 \\ \hline & 1 & -2 & 5 & -8 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & -2 & 5 & -8 & 4 \\ & & 1 & -1 & 4 & -4 \\ \hline & 1 & -1 & 4 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 \\ & & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$(x+2)(x-1)(x-1)(x^2 + 4) = 0$$

$$x = -2 \quad x = 1 \text{ w/ mult. 2}$$

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm 2i$$

complex zeros occur in conjugate pairs

Let $f(x)$ be a polynomial that has real coefficients. If $a+bi$ ($b \neq 0$), is a zero of $f(x)$, then the conjugate $a-bi$ is also a zero of $f(x)$

Example

Write a third degree polynomial with zeros of -5 and $3-2i$, $3+2i$

$$f(x) = (x+5)(x-(3-2i))(x-(3+2i)) = f(x)$$

Write a 4th degree polynomial with zeros -1 , -1 , and $3i$, $-3i$

$$(x+1)^2(x-3i)(x+3i)$$

Examples

Find all the zeros of the function.

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60 \quad \text{given that } 1+3i \text{ is a zero.}$$

$$(x - (1+3i))(x - (1-3i)) \left(\begin{array}{c} 1-3i \\ \end{array} \right)$$

$$(x - 1 - 3i)(x - 1 + 3i)$$

$$\cancel{x^2} - \cancel{x} + 3ix - \cancel{x} + 1 - 3i - 3ix + 3i - 9i^2$$

$$\boxed{3, -2, 1-3i, 1+3i}$$

$$(x^2 - 2x + 10)(x^2 - x - 6) = 0$$

$$\text{Zero} = 0$$

$$(x - a)$$

$$(x - 3)(x + 2) = 0$$