Chapter 1: Section 6: Inverse Functions

Let f and g be two functions such that $f(g(x)) = x$, for every x in the domain of g AND $g(f(x))= x$ for every x in the domain of f.

If this happens, then g is the inverse of f, denoted by $f^{-1}$ "f inverse"

To algebraically verify two functions f and g are inverses, you need to show both $f(g(x))= x$ and $g(f(x))=x$

$f(g(x)) = (x-2)+2 = x$ $\quad f(x) = x+2$

$g(f(x)) = (x+2)-2 = x$ $\quad g(x) = x-2$

1) $f(g(x)) \stackrel{?}{=} x$

$$\frac{(4x+9)-9}{4}$$

$$\cancel{\frac{4 \cdot x}{4 \cdot 1}}$$

$$x$$

$g(f(x)) \stackrel{?}{=} x$

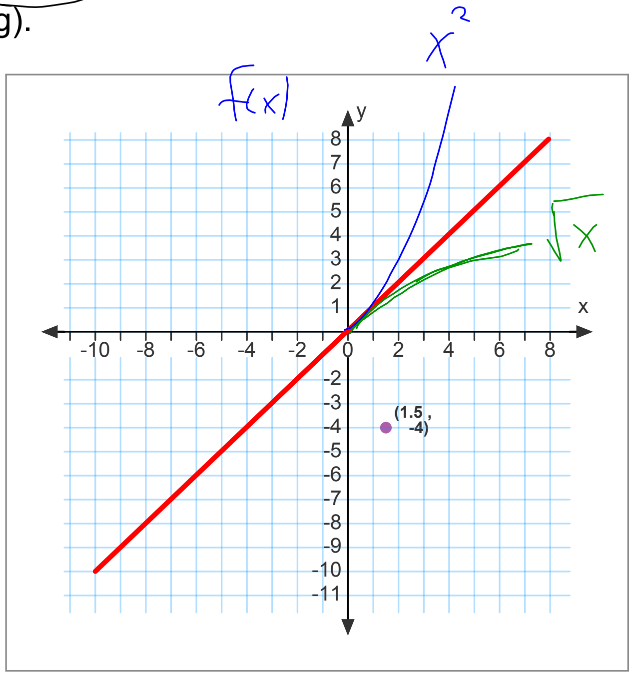
$$\cancel{4}\left(\frac{x-9}{\cancel{4}}\right) + 9$$

$$x - 9 + 9$$

$$x$$

If two functions f and g are inverses, then the graphs of f and g will be symmetric with respect to the line y = x
(reflect f over y=x and you will get g).

$(x, y) \rightarrow (y, x)$




1-1

Not all functions have inverses.

A function f is <u>one to one</u> if, for a and b in its domain, f(a) = f(b) implies that a = b

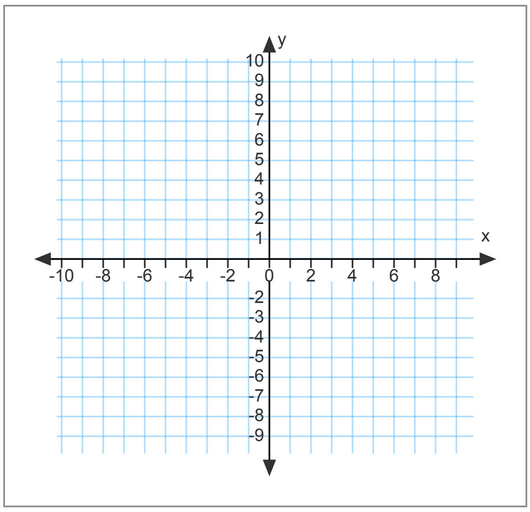
** one x-value is matched to one y-value AND one y-value is matched to one x-value

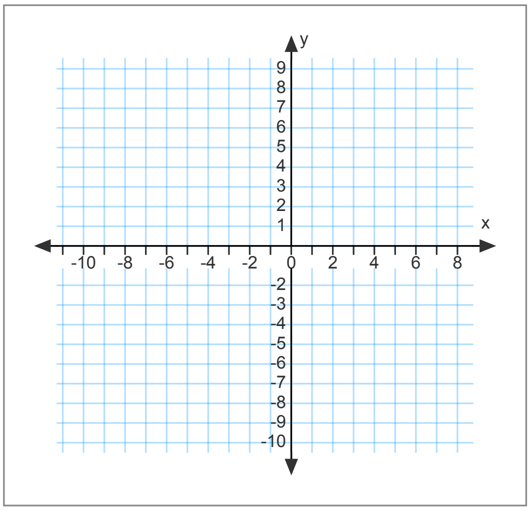
Vertical Line Test

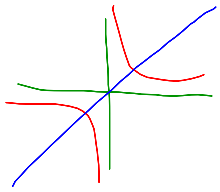
Horizontal Line Test - Inverse exists?

A function f has an inverse function $f^{-1}$ if and only if f is a one to one function.

Horizontal line test: If any horizontal line crosses a graph at more than one point then the function does NOT have an inverse.





To find an inverse algebraically:

1. Use horizontal line test to determine if an inverse exists
2. Replace f(x) with y
3. Switch the variables x and y
4. Solve for the y
5. Replace y with $f^{-1}$ (x)
6. Verify f( $f^{-1}$ (x)) = $f^{-1}$ (f(x)) = x

$f(x) = \frac{4}{x}$

$y = \frac{4}{x}$

$y \cdot x = \frac{4}{\cancel{x}} \cdot \cancel{x}$

$\frac{y \cdot x}{x} = \frac{4}{x}$

$y = \frac{4}{x}$

$f^{-1} = \frac{4}{x}$

$f(f^{-1}(x)) = \frac{4}{\left(\frac{4}{x}\right)} = 4 \cdot \frac{x}{4}$

$= x$

$f^{-1}(f(x)) = \frac{4}{\frac{4}{x}}$