Precalculus 2.2

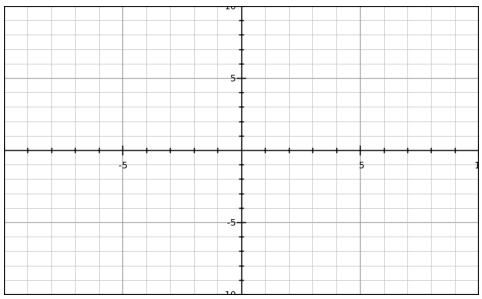
Polynomials of Higher Degree

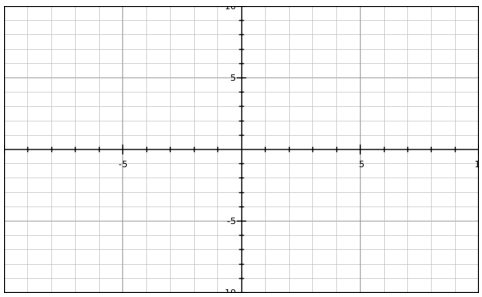
polynomials $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ graphs are continuous

a_n is the leading coefficient and $\neq 0$ *n* is the degree of the polynomial and is an integer

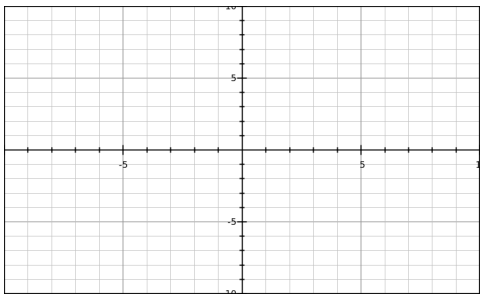
Properties of Polynomial Functions

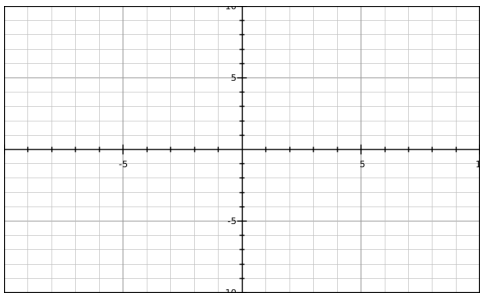
1) polynomial of degree *n* can have at most *n-1* turning points





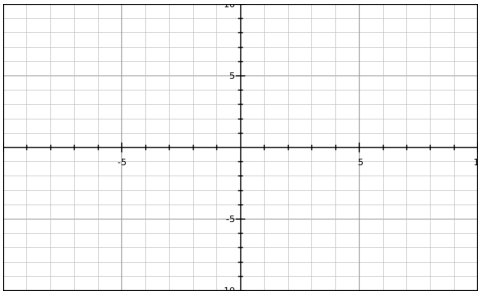
2) If the polynomial is degree *n*, then there is at most *n* real zeros.



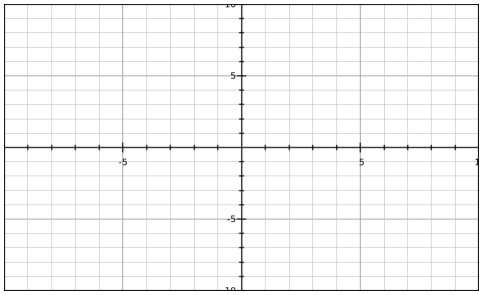


3) polynomial with even degree, both ends go up or both ends go down
polynomial with odd degree, one end goes up and the other goes down

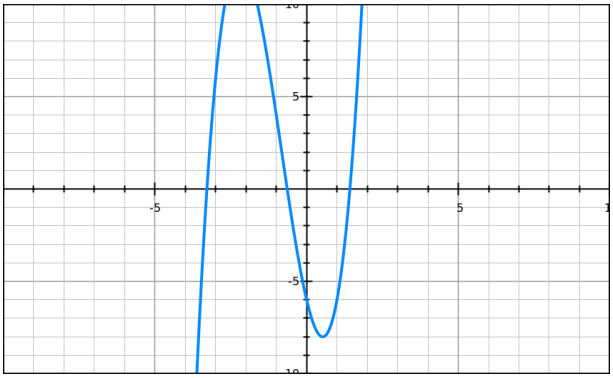
even degree

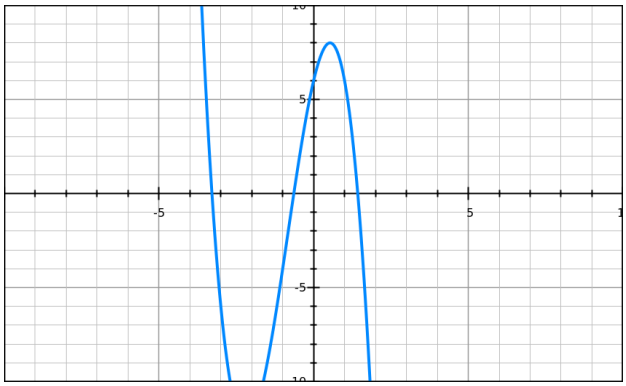


odd degree



4) If the graph goes up as *x* becomes large, the leading coefficient must be positive. If the graph goes down as *x* becomes large, the leading coefficient is negative





If x = a is a zero, then x=a is a solution to f(x) AND (x - a) is a factor of f(x).

If (x - a) is a factor of f(x) , then x = a is a zero

$(t-3)(t-3)=0$

$t=3$ multiplicity of 2

For a polynomial, with factor $(x - a)^k$, $k > 1$ gives a repeated zero $x = a$ of <u>multiplicity</u> k

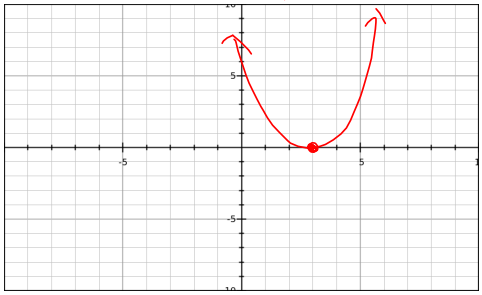
- If k is odd, the graph crosses the x-axis at x = a
- If k is even, the graph "touches" the x-axis (but does not cross) at x = a

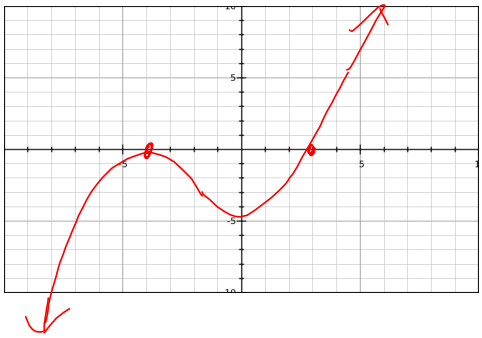
degree 3

$(t-3)^2 = 0$

$t = 3$ m2

$(x-3)^1(x+4)^2 = 0$





$$f(x) = x(x-6)(x+3)^2 = 0$$

$x+3=0$

$x=-3$

$x-6=0$

$x=6$

For the above function, find the following...

$x^2 \cdot x^2$

- each zero and its multiplicity,

$x=0$ crosses

$x=-3$ mult. 2 Touches

$x=6$ crosses

$a_n x^n + a_{n-1} x^{n-1}$

- does the graph cross or touch the x-axis at each zero,

$x^2 \cdot x = x^3 \cdot x$

$1x^4$

- maximum number of possible turning points,

3

- end behavior of the graph.

Left end: UP

Right end: UP

Intermediate Value Theorem: Let a and b be real numbers, such that a < b.
If f is a polynomial function such that f(a) does not equal f(b)
then in the interval $a \leq x \leq b$, f takes on every value between f(a) and f(b)