What's the remainder? (no calculator)

When 12 is divided by 5?

$12 = 5 \cdot 2 + 2$

When 27 is divided by 3?

$27 = 3 \cdot 9 + 0$

When 1349 is divided by 4?

$$
\begin{array}{r}
337 \\
4\overline{)1349} \\
-12 \\
\hline
14 \\
-12 \\
\hline
29 \\
-28 \\
\hline
1
\end{array}
$$

2.3 Finding the Other Zeros of Polynomials

If you know a zero of a polynomial, then you know a factor of the polynomial.

If you have a factor, you can use division to find the other zeros.

If f(x) and d(x) are polynomials such that d(x) does not equal 0 and the degree of d(x) ≤ degree of f(x)

there exist unique polynomials q(x) and r(x) such that:

f(x) = d(x) q(x) + r(x)

dividend = (divisor)(quotient) + remainder

Where r(x) = 0 or the degree of r(x) < degree of d(x).

If the remainder r(x) = 0, then d(x) and q(x) are factors of f(x)

Use long division only when you have to. $4x^3-7x^2-11x+5 = (4x+5)(x^2-3x+1)$

try #2 on homework sheet

$q(x) = x^2-3x+1$

$$
\begin{array}{r}
x^2-3x+1 \\
4x+5\overline{)\,4x^3-7x^2-11x+5} \\
-(4x^3+5x^2) \\
\hline
-12x^2-11x+5 \\
-(-12x^2-15x) \\
\hline
4x+5 \\
-(4x+5) \\
\hline
0
\end{array}
$$

$r(x)=0$

There is a quicker way to divide **IF** you are dividing by a function of the form $x-k$

$x-4$ $x+5$ $x--5$ -5

$p(x)=ax^3+bx^2+cx+d$ $d(x)=x-k$

$\frac{p(x)}{d(x)}$ can be done using synthetic division (must use place holders)

k | a b c d

4

x^3+5x-7

1 0 5 -7

find $\dfrac{5x^3+18x^2+7x-6}{x+3} = (x+3)(5x^2+3x-2)$ $q(x) = 5x^2+3x-2$

$-3 \mid 5 \quad 18 \quad 7 \quad -6$

$\quad\quad -15 \quad -9 \quad 6$

$5 \quad 3 \quad -2 \quad 0 = r(x)$

$5(-3)^3+18(-3)^2+-21-6$

$-135+162-27$

$-162+162=0$

$= f(-3)$

$$\frac{5x^3+6x+8}{x+2}$$

$-2 \mid 5 \quad 0 \quad 6 \quad 8$

$\quad\quad -10 \quad 20 \quad -52$

$5 \quad -10 \quad 26 \quad -44 = r(x)$

$q(x) = 5x^2-10x+26$

Remainder Theorem: If a polynomial f(x) is divided by x - k, the remainder is r = f(k)

Factor Theorem: A polynomial f(x) has a factor x - k, if and only if f(k) = 0 = r

Given $x+3$ is a factor of $f(x)=3x^3+2x^2-19x+6$

find all the zeros of *f(x)*

$=(x+3)(3x^2-7x+2)$

$0=(x+3)(3x-1)(x-2)$

$-3\,|\,3\quad 2\quad -19\quad 6$

$\qquad -9\quad 21\quad -6$

$3\quad -7\quad 2\quad 0$

Zeros

$-3, \frac{1}{3}, 2$

Find all the factors of $f(x)=4x^4-55x^2-45x+36$

$4-55-45+36$

$f(x)=(x+3)(4x^3-12x^2-19x+12)=(x+3)(x-4)(4x^2+4x-3)$

$-3\,|\,4\quad 0\quad -55\quad -45\quad 36$

$\qquad -12\quad 36\quad 57\quad -36$

$4\quad -12\quad -19\quad 12\quad 0$

$4\,|\,4\quad -12\quad -19\quad 12$

$\qquad 16\quad 16\quad -12$

$4\quad 4\quad -3\quad 0$

$f(x)=(x+3)(x-4)(2x+3)(2x-1)$

$210 = 2 \cdot 105 = 2 \cdot 5 \cdot 21$

$2 \cdot 5 \cdot 3 \cdot 7$

Prime Factors

$\frac{210}{2} = 105$

$210 = 2 \cdot \frac{105}{5}$

$210 = 2 \cdot 5 \cdot 21$