

## Precalculus Sec. 2.6

### Rational Functions

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-a rational function can be written in the form  $f(x) = \frac{N(x)}{D(x)}$   
where  $N(x)$  and  $D(x)$  are polynomials

- the domain of a rational function includes all real numbers  $x$  except the values of  $x$  that make the denominator equal to zero.

#### Examples

Identify the domain of the following functions.

$$f(x) = \frac{3x-6}{8-5x}$$

$$D: \mathbb{R}, x \neq \frac{8}{5}$$

$$\begin{aligned} 8-5x &= 0 \\ 8 &= 5x \\ \frac{8}{5} &= \frac{5x}{5} \\ \frac{8}{5} &= x \end{aligned}$$

$$g(x) = \frac{4}{x^2 + 12x - 45}$$

$$D: \mathbb{R}, x \neq -15, 3$$

$$\begin{aligned} x^2 + 12x - 45 &= 0 \\ (x+15)(x-3) &= 0 \\ x &= -15 \quad x = 3 \end{aligned}$$

### Definitions of Asymptotes and Holes

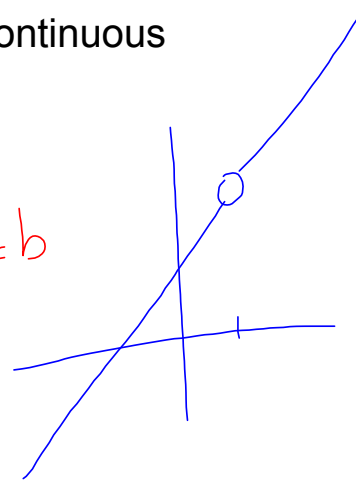
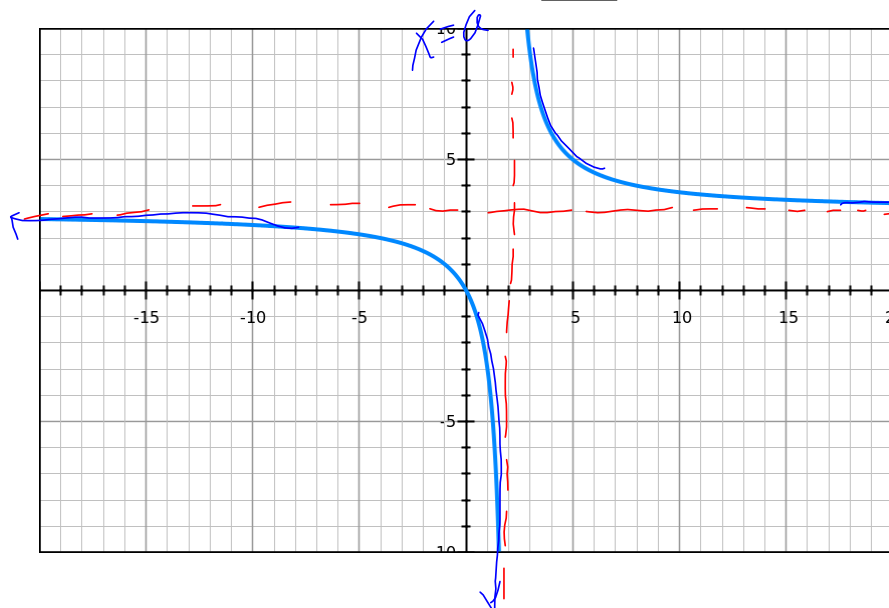
-The line  $x=a$  is a vertical asymptote of the graph of  $f$  if

$f(x) \rightarrow \infty$  or  $f(x) \rightarrow -\infty$  as  $x \rightarrow a$ , either from the right or from the left

-The line  $y=b$  is a horizontal asymptote of the graph of  $f$  if

$f(x) \rightarrow b$  as  $x \rightarrow \infty$  or  $x \rightarrow -\infty$

- a function is undefined at a hole, but is otherwise continuous



## Finding the Horizontal and Vertical Asymptotes and Holes

$$f(x) = \frac{N(x)}{D(x)}$$

factors

- **the function will have a vertical asymptote** at the zeros of the denominator (zeros of the denominator ONLY)
- **the function will have a horizontal asymptote...**
  - if the degrees of the numerator and denominator are the same, then the horizontal asymptote is at the y-value equal to the fraction of the leading coefficients
  - if the degree of the denominator is greater than the degree of the numerator, then the horizontal asymptote is at  $y=0$

factor

**the function will have a hole** at the x-value that is a zero of both the numerator and denominator

- *if the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote.*

**Examples** Identify the horizontal, vertical asymptotes and holes for the given functions.

$$f(x) = \frac{5}{3x-6}$$

$$3x-6=0$$

$$x=2$$

$$\text{H.A.: } y=0$$

$$\text{V.A.: } x=2$$

$$\text{Hole: } \underline{\hspace{2cm}}$$

$$f(x) = \frac{5x-10}{3x-6}$$

$$(4x-1)(x+1)$$

$$f(x) = \frac{4x^2+3x-1}{3x^2-27}$$

$$3(x^2-9)$$

$$3(x+3)(x-3)$$

$$\text{H.A.: } y = \frac{5}{3}$$

$$\text{V.A.: } \underline{\hspace{2cm}}$$

$$\text{Hole: @ } x=2$$

$$\text{H.A.: } y = \frac{4}{3}$$

$$\text{V.A.: } x=-3 \text{ \& } x=3$$

$$\text{Hole: } \underline{\hspace{2cm}}$$