Precalculus Sec 3.2 Logarithmic Functions

- a logarithmic function is the inverse of an exponential function

DEF. For $x>0$, $a>0$, and $a \neq 1$

$y = \log_a x$ if and only if $a^y = x$

For example, $\log_2 8 = 3 \longrightarrow 2^3 = 8$

$\log_a x$ reads "log base a of x"

Examples

$\log_4 64 = 3$

$4^x = 64$

$\log_3 81 = 4$

$3^x = 81$

$\log_6 x = 5$

$6^5 = x$

7776

$\log_x 125 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{125}$

$x = 5$

- The base of a logarithm can be any positive number except 0 and 1, there are two that are used the most, base 10 and base *e*.

$\log_{10} x$ ⟶ we don't need to write the base 10, it is implied ⟶ $\log x$

$\log_e x$ ⟶ a log base *e* is called the natural logarithm and written as ⟶ $\ln x$

Properties of logarithms with any allowed base

1) $\log_a 1 = 0$ $a^0 = 1$

2) $\log_a a = 1$ $a^1 = a$

3) $\log_a a^x = x$ $a^x = a^x$

4) If $\log_a x = \log_a y$ then $x = y$

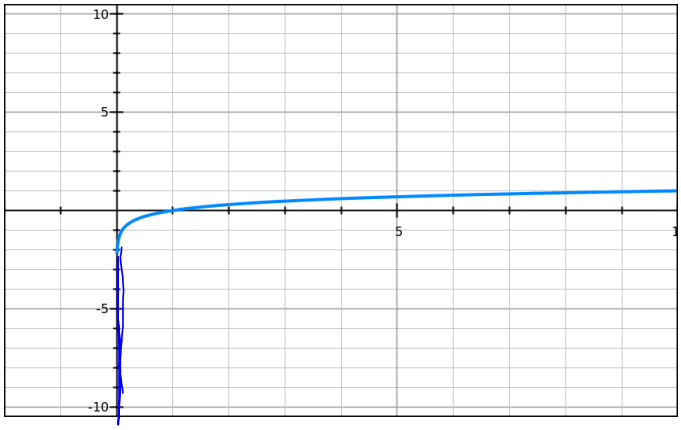
$\log_4 4^2 = 2$

$\ln e^3 = 3$

End Day 1

Sec. 3.2 Day 2

The graph of $y = \log_a x$ comes from the fact that logarithms are inverses with $y = a^x$



$f(x) = \log_a x$ $\log_2 X$

Domain: x>0
Range: *all real*
x-intercept: *x=1*
increasing throughout domain
V.A.: *x=0*

Transformations - same rules as before for shifts, reflections

EXAMPLE

$x-3>0$
$x>3$

Parent: $f(x)=\log_2 x$

$g(x)=\log_2(x-3)+4$ is the parent function after being...

shift right 3

shift up 4

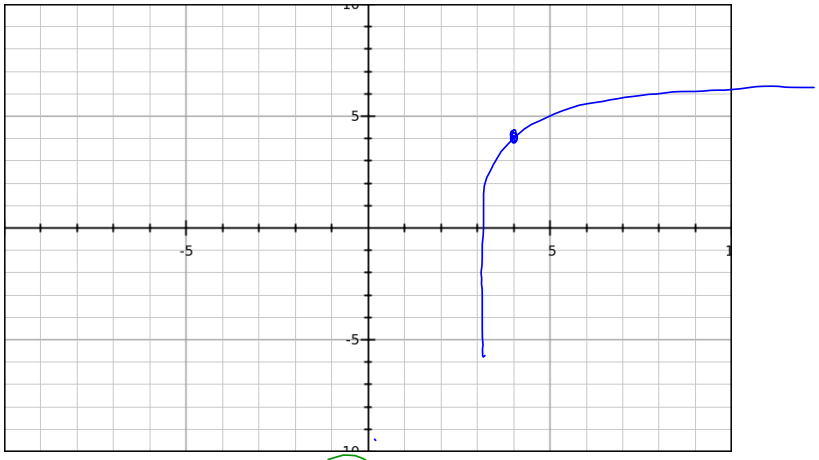
- When graphing the logarithm function, concentrate on the vertical asymptote for the shifts left and right, and the x-intercept for the shifts up and down.

Sketch g(x) from above.

D: $x>3$

V.A: $x=3$

x-int: $3\frac{1}{16}$



$0=\log_2(x-3)+4$

$-4=\log_2(x-3)$

$2^{-4}=x-3$

$\frac{1}{2^4}=\frac{1}{16}=x-3$

$x=\frac{1}{16}+3$

$x=3\frac{1}{16}$

Finding Domain of logarithmic functions

You can not take the log of a negative number or of zero.

EXAMPLE

Find the domain, x intercept and the vertical asymptote of

$f(x) = \ln(3x - 7)$

Domain: $3x - 7 > 0$

$3x > 7$

$x > \frac{7}{3}$

VA: Same as domain #

$x = \frac{7}{3}$

x-int

$0 = \ln_e(3x - 7)$

$e^0 = 3x - 7$

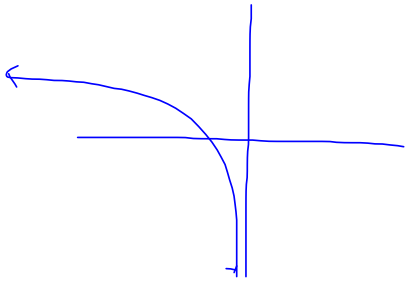
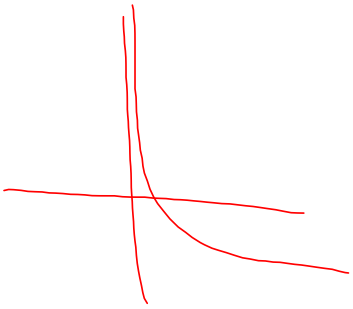
$1 = 3x - 7$

$8 = 3x$

$\frac{8}{3} = x$

$y = \log_a(-x)$ $\Rightarrow$ reflect over y-axis

$y = \log_a(-x)$



$y = -\log_a x$ $\Rightarrow$ reflect over x-axis