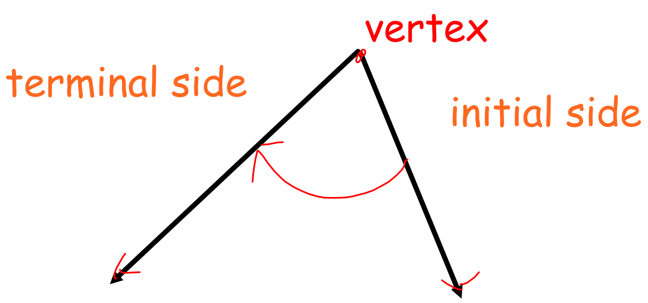
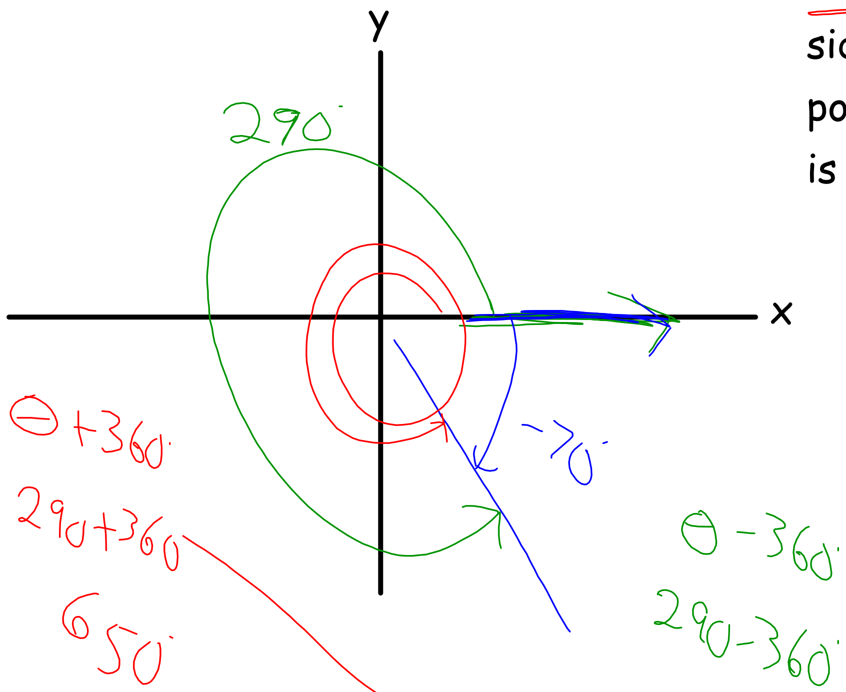
Chapter 4 - Trigonometry

4.1 - Radian and Degree Measure





standard position- initial side of angle is on the positive x-axis and vertex is at the origin

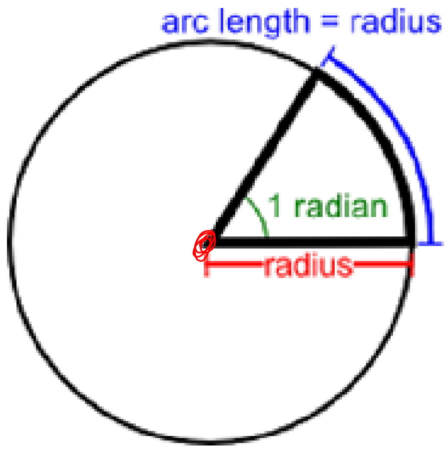
positive angles go counterclockwise

negative angles go clockwise

coterminal angles are different angles that have the same terminal side

what are two coterminal angles (+ & -) with 135?

Radian - a unit of angle measure. One radian (1 rad) is the central angle that intercepts an arc equal in length to the radius of the circle



theta

$$\theta = 1 rad$$

$$C = 2\pi r$$

1 revolution is 2π radians

$360° = 2\pi$ rad.

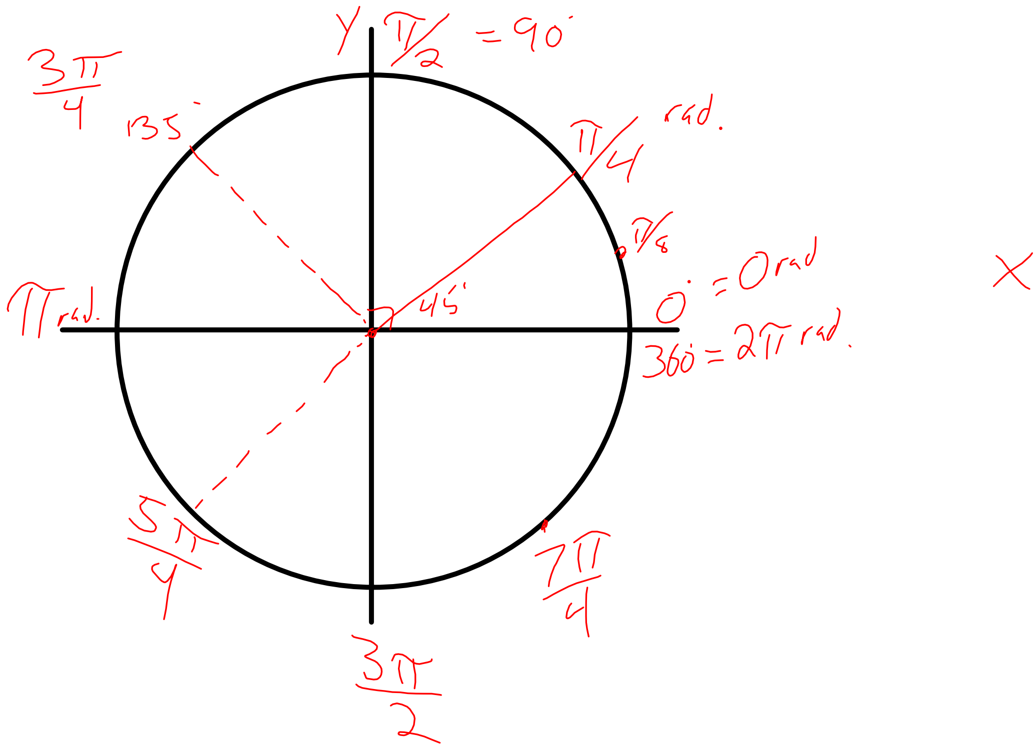
$\boxed{180° = \pi \text{ rad}}$

1/2 revolution = π rad

1/4 revolution = $\frac{\pi}{2}$ rad.

1/6 revolution = $\frac{\pi}{3}$ rad.

$\frac{2\pi}{6} = \frac{\pi}{3}$



Y
π/2 = 90°
3π/4
135°
π/4 rad.
π/8
45°
π rad.
0° = 0 rad
360° = 2π rad.
X
5π/4
7π/4
3π/2

Converting between Degrees & Radians

- one way that always works is using proportions

$$\frac{\pi}{180} = \frac{rad}{deg}$$

$$\frac{150}{1} \cdot \frac{\pi}{180} = \frac{\theta}{\cancel{150}} \cdot 150$$

Example

150°

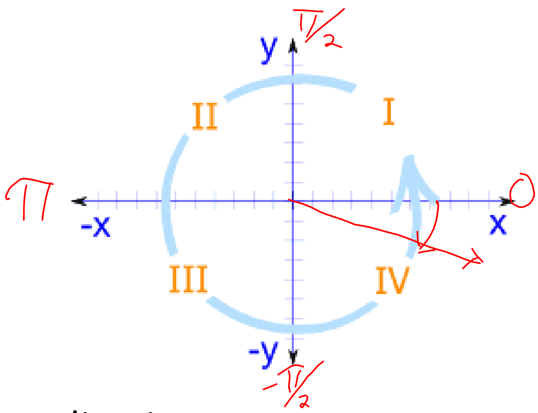
$$\frac{150\pi}{180} = \theta = \frac{5\pi}{6} \text{ rad}$$

2.617 rad

when going from radian (in terms of π), use the fact that $\pi = 180°$

$$\frac{3\pi}{5} = \frac{3(180)}{5} = \frac{540}{5} = 108°$$

Quadrants - areas of the coordinate grid



Determine what quadrant each angle is ending in.

$123°$ II

$\frac{\pi}{3}$ $\frac{180}{3}=60°$ I

$\frac{4\pi}{5}$ II

$-240°$ II

$-\frac{\pi}{12}$ IV

Determine two coterminal angles for each. (one positive, one negative)

$\frac{\pi}{3}$

$\frac{4\pi}{5}$

complementary angles - two positive angles that add to 90° or $\frac{\pi}{2}$

supplementary angles - two positive angles that add to 180° or π

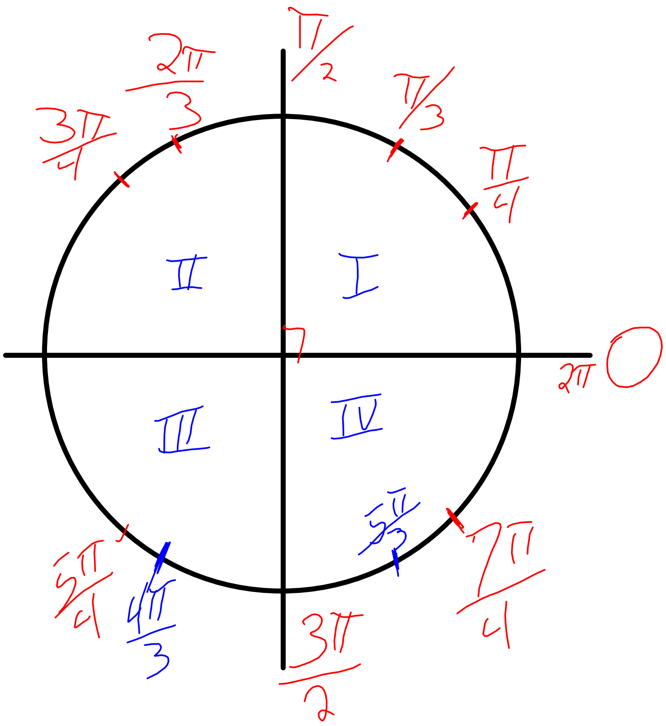
comp: ___

$$\frac{4\pi}{5} + \theta = \pi$$

$$-\frac{4\pi}{5} \qquad\qquad -\frac{4\pi}{5}$$

$$\theta = \frac{\pi}{1} - \frac{4\pi}{5}$$

$$\frac{5\pi}{5} - \frac{4\pi}{5} = \frac{\pi}{5}$$



$\frac{4\pi}{3}$

III

$\frac{3\pi}{3}$

$\frac{\pi}{6} = \frac{180^{\circ}}{6}$

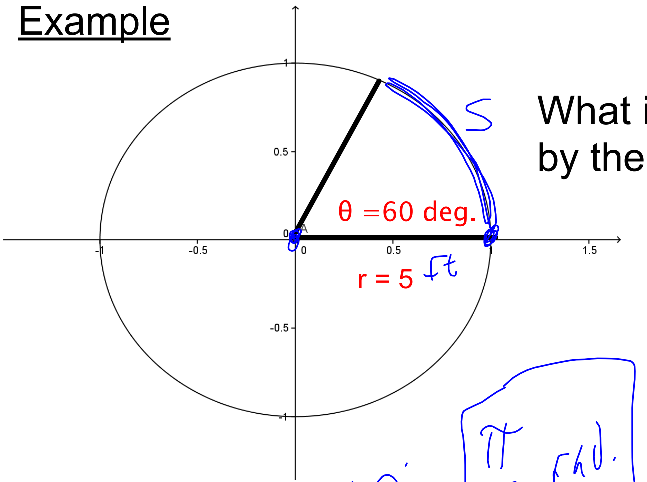
30°

$\frac{\pi}{3}$

<u>arc length</u> - distance around a circle $s = r\theta$

s = arc length r = radius θ = central angle measured in radians

<u>Example</u>



What is the length of the arc intercepted by the angle?

$s = 5 \cdot \frac{\pi}{3} = \boxed{\frac{5\pi}{3}\text{ ft}}$

$60^\circ = \boxed{\frac{\pi}{3}\text{ rad.}}$

With this equation, $s = r\theta$ you can be given any two pieces of information and find the third.

Motion of a Partical Moving @ a Constant Speed Around a Circle

radian

linear speed = $\dfrac{\textit{arclength}}{\textit{time}} = \dfrac{s}{t} = \dfrac{r\theta}{t}$

linear speed measures how fast a particle moves

angular speed = $\dfrac{\textit{centralangle}}{\textit{time}} = \dfrac{\theta}{t}$

angular speed measures how fast the angle changes

My bike tire has a diameter of 30". If it rotates 12 times per second, how fast is going?

radians

$$\text{Linear speed} = \frac{s}{t} = \frac{r\cdot\theta}{t} = \frac{15'' \cdot \boxed{50\pi}}{1}$$

750π

$\theta = 2\pi \cdot 25$

$$\frac{2356.19\ \cancel{in}}{1\ \cancel{sec}}\cdot\frac{1\ \cancel{ft}}{12\ \cancel{in}}\cdot\frac{1\ mile}{5280\ \cancel{ft}}\cdot\frac{60\ \cancel{sec}}{1\ \cancel{min}}\cdot\frac{60\ \cancel{min}}{1\ Hr} = \boxed{133.8\ \frac{M}{H}}$$