Sec. 4.7
Inverse Trig Functions

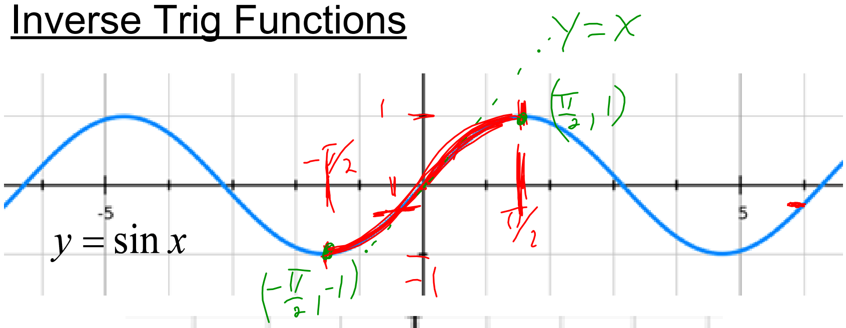
- If a function is to have an inverse, then it must pass the Horizontal Line Test; sine, cosine, and tangent all fail. This means the domains of these functions must be restricted in such a way to include all the values in the range and include the acute angles found in the right triangles.

Inverse Trig Function Notation

$y = \sin^{-1} x$ or $y = \arcsin x$ y = angle measure

$y = \sin x$

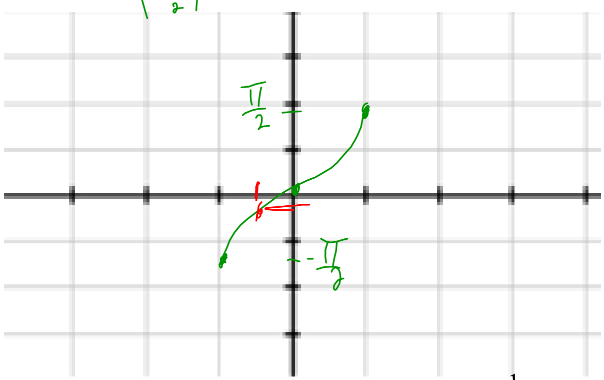
Inverse Trig Functions



$$y = \sin^{-1} x$$

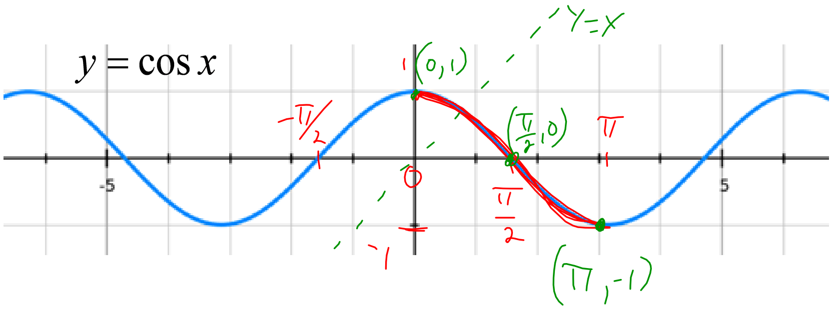
Domain: $-1 \le x \le 1$

Range: $\boxed{-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$



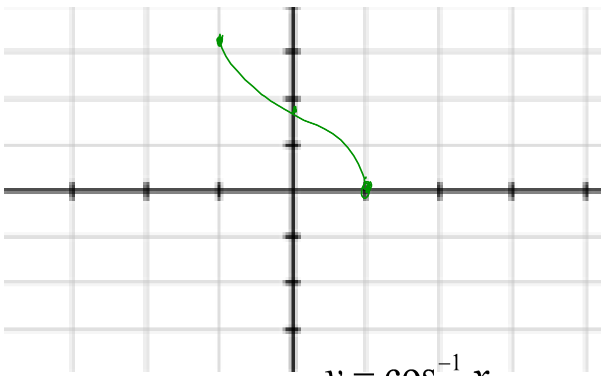
$y = \sin^{-1} x$

$$y = \cos^{-1} x$$



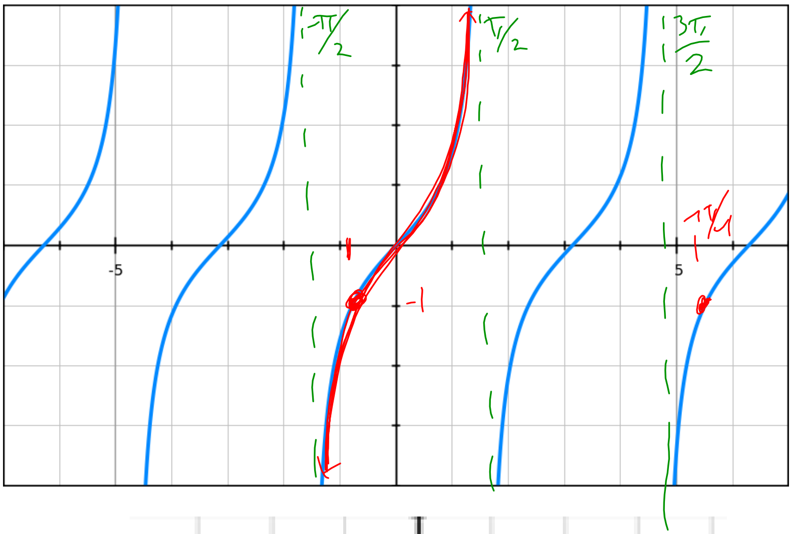
Domain: $-1 \le x \le 1$

Range: $0 \le \theta \le \pi$



$y = \tan x$

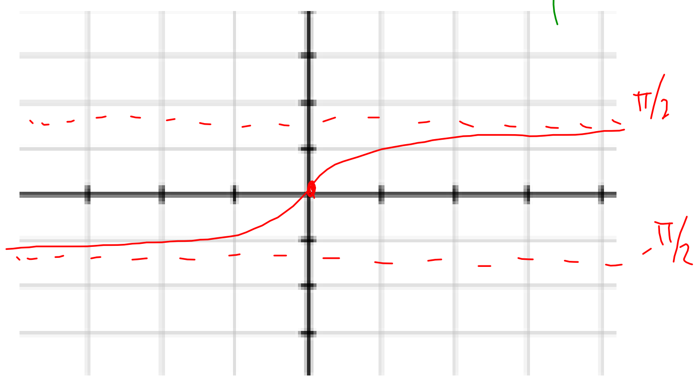
$\frac{\text{Sin}}{\text{Cos}}$



$y = \tan^{-1} x$

Domain: $\mathbb{R}$ (all real)

Range: $\boxed{-\frac{\pi}{2} < \theta < \frac{\pi}{2}}$

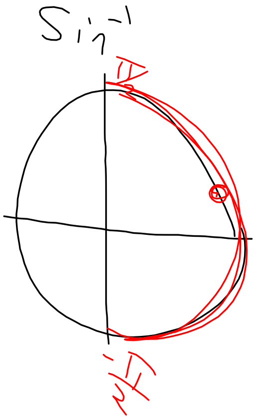


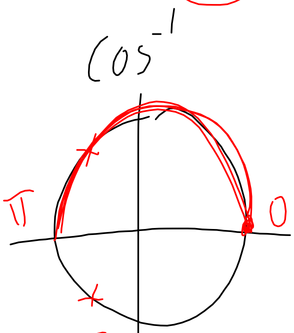
Examples of Inverse Trig.

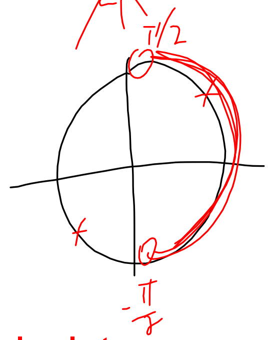
$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ~~$\frac{5\pi}{6}$~~

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ ~~$\frac{5\pi}{4}$~~

$\arctan 1 =$ $\tan^{-1}(1) = \frac{\pi}{4}$ ~~$\frac{5\pi}{4}$~~





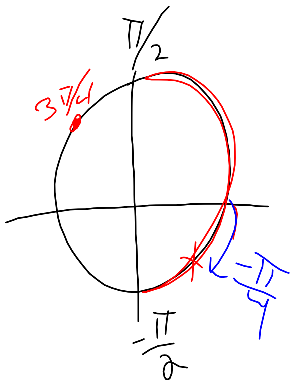


- if the number is not on the unit circle, then use the calculator.

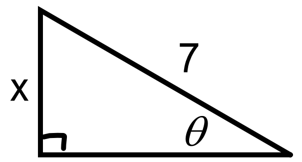
$\frac{7\pi}{4}$ ~~$\frac{3\pi}{4}$~~

$\tan\theta = -1$

$\tan^{-1}(-1) = -\frac{\pi}{4}$



Example Write the equation for θ in terms of x.



$$\sin\theta = \frac{x}{7}$$

$$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{x}{7}\right)$$

$$\boxed{\theta = \sin^{-1}\left(\frac{x}{7}\right)}$$

<u>Compositions of Functions</u> - inverse trig. functions can, but not always, cancel each other out.

Follow order of operations (inner most parentheses first)

$$\sin\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) =$$

$$\sin\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cos\left(\sin^{-1}\frac{1}{2}\right) =$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\cos\frac{\pi}{3}\right) =$$

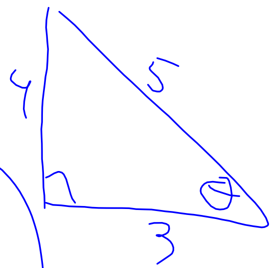
$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) =$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$

$$\tan\left(\sin^{-1}\left(\frac{4}{5}\right)\right) =$$

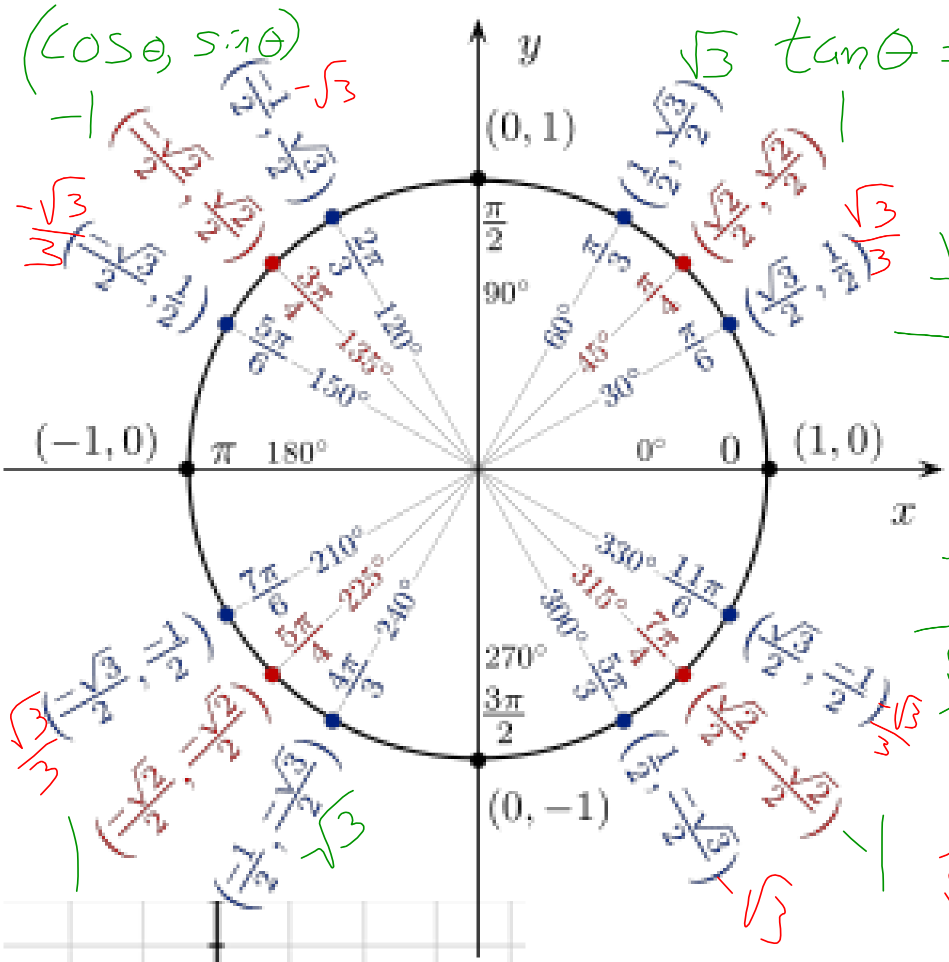
θ



$$\tan\theta = \frac{4}{3}$$

$(\cos\theta, \sin\theta)$

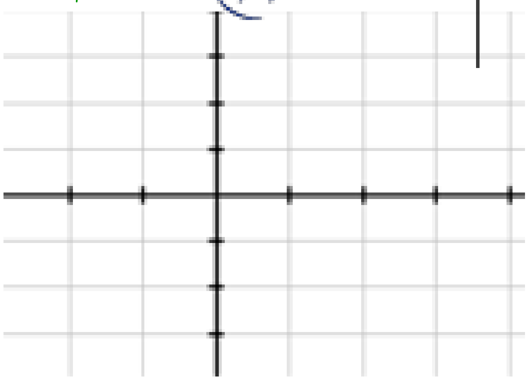
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad \dfrac{y}{x}$



$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$$



$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \qquad \cancel{\frac{7\pi}{6}} \qquad \frac{11\pi}{6}$$