Precalculus Chapter 4 Review

KEY

1.) Find a positive and a negative angle that is coterminal with $\frac{4\pi}{3}$. $\pm\frac{6\pi}{3}$

1.) positive $\frac{10\pi}{3}$

negative $-\frac{2\pi}{3}$

2.) Find the complement and supplement of the angle $\frac{\pi}{5}$

$\frac{\pi}{5}+C=\frac{\pi}{2}$

$C=\frac{5\pi}{10}-\frac{2\pi}{10}=\frac{3\pi}{10}$

$\frac{\pi}{5}+S=\pi$

$S=\frac{4\pi}{5}$

2.) comp. $\frac{3\pi}{10}$

supp. $\frac{4\pi}{5}$

3.) If the angle is in radians, convert it to degrees. If the angle is in degrees, convert it to radians.

a.) $\frac{2\pi}{9}=\frac{2\cdot 180}{9}=40$

b.) $405°$

$\frac{\pi}{180}=\frac{x}{405}$

$\frac{405\pi}{180}=x=\frac{81\pi}{36}=\frac{9\pi}{4}$

3.) a.) $40°$

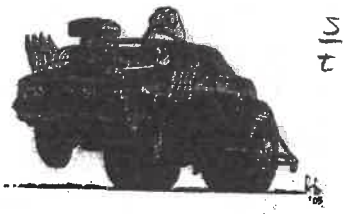
b.) $\frac{9\pi}{4}$

4.) Find the length of the arc on a circle with a radius of 14 cm intercepted by a central angle of $\frac{7\pi}{6}$.

$S=\frac{7\pi}{6}\cdot 14$

4.) $\frac{98\pi}{6}$ or $\frac{49\pi}{3}$ cm

5.) Charlie's 1981 Dodge K-car has a broken speedometer. With the use of a high speed camera he has determined that his Mickey Thompson tires with a 15 inch radius rotates 30 times every second. How fast is the K-car going in MPH? (5280 feet = 1 mile)



$\frac{S}{t}=\frac{15''\cdot 2\pi\cdot 30}{1\,sec}=\frac{900\pi\ in}{sec}\left|\frac{1\,ft}{12\,in}\right|\frac{1\,mile}{5280\,ft}\left|\frac{60\,sec}{1\,min}\right|\frac{60\,min}{1\,Hr}$

5.) 160.65 MPH

6.) Use the unit circle to give the exact value of the given function.

a.) $\sin\frac{11\pi}{6}$ b.) $\cos\frac{5\pi}{6}$ c.) $\tan\frac{-\pi}{4}$ d.) $\sec\frac{7\pi}{3}$

$\frac{1}{\cos\frac{2\pi}{3}}=\frac{1}{-1/2}$

6.) a.) $-\frac{1}{2}$

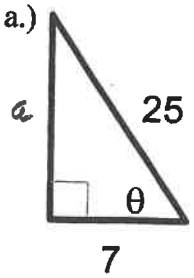
b.) $-\frac{\sqrt{3}}{2}$

c.) -1

d.) -2

7.) Find the exact value of the given trigonometric functions of the angle θ.

a.)



$a^2+7^2=25^2$

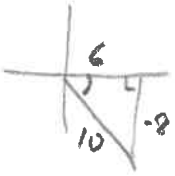
$a=24$

$\sin\theta=\frac{24}{25}$

$\csc\theta=\frac{25}{24}$

$\cos\theta=\frac{7}{25}$

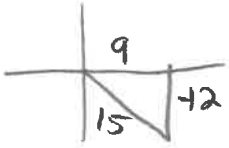
8.) The point $(6,-8)$ is on the terminal side of an angle in standard position. Determine the exact values of the given trigonometric functions of the angle.



$\tan\theta =$ $\frac{-8}{6}$

$\sec\theta =$ $\frac{10}{6}$

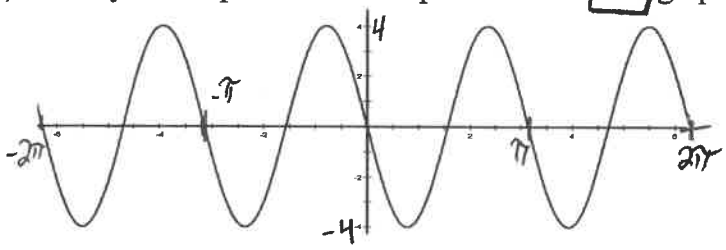
9.) *If* $\cos\theta = \frac{9}{15}$ *and* θ *is in Quadrant IV, then find the exact values of the following.*



$\sin\theta =$ $\frac{-12}{15}$

$\tan\theta =$ $\frac{-12}{9}$

10.) Identify the amplitude and the period of this sine graph. $a=4$ $b=2$



$PS = \frac{\pi}{2} = \frac{c}{b}$

$\frac{\pi}{2} = \frac{c}{2} \rightarrow c = \pi$

$d = 0$

10.) amp. 4

per. π

equation

$y = 4\sin(2\theta - \pi)$

↑ could be + also

11.) Identify the amplitude, period and phase shift of the function.

a.) $y = -2\sin\left(4x - \frac{\pi}{2}\right)$ b.) $y = 5\cos(3x + \pi)$

$\frac{\frac{\pi}{2}}{4}$

11.) a. amp. 2

per. $\pi/2$

phase shift $\pi/8$

b.) amp. 5

per. $2\pi/3$

phase shift $-\pi/3$

12.) Identify the period, phase shift, relative minimum and relative maximum for the equation $y = 3\sec(6x - \pi)$.

period $\frac{2\pi}{6} = \pi/3$

phase shift $\pi/6$

rel. min. 3

rel. max. -3

13.) Evaluate the following expressions.

a.) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ b.) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ c.) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

a.) $\pi/4$ b.) $\pi/3$ c.) $5\pi/6$

In the lines provided, name the amplitude, period, horizontal center and phase shift for each of the following. Then sketch one period of the graph.

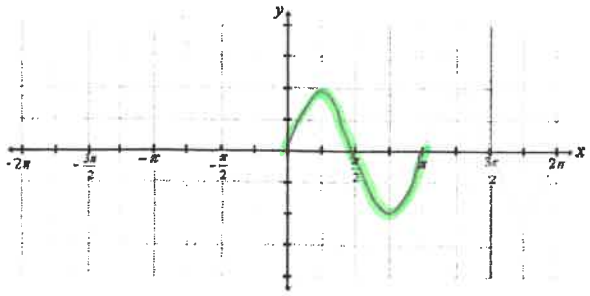
1) $y = 2 \sin(2x)$

amplitude = 2

period = π

phase shift = 0

vertical shift = 0



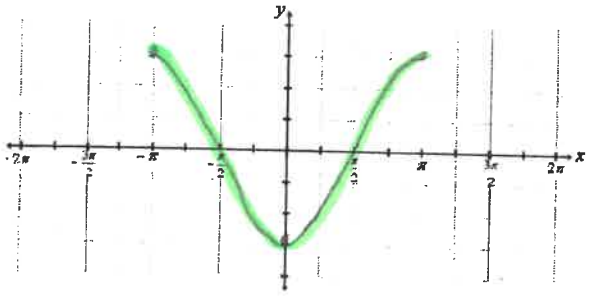
2) $y = 3 \cos(x + \pi)$

amplitude = 3

period = 2π

phase shift = $-\pi$

vertical shift = 0



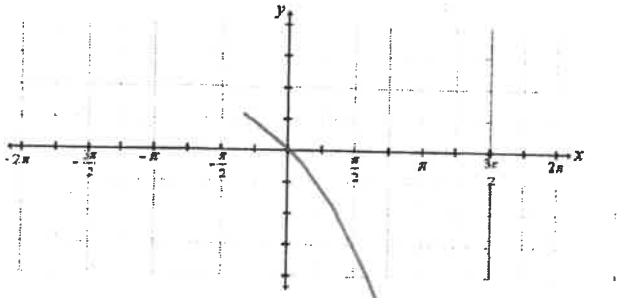
3) $y = 4 \cos(3x) - 6$

amplitude = 4

period = $\frac{2\pi}{3}$

phase shift = 0

vertical shift = -6



Write the equation containing sin that satisfies the given information:

4). Amplitude = 3 Period = 3π

$\frac{2\pi}{b} = 3\pi$

$b = \frac{2}{3}$

$y = 3\sin\left(\frac{2}{3}\theta\right)$

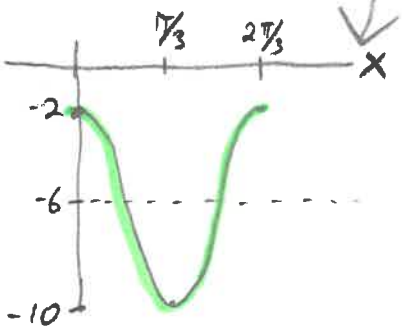
5). Amplitude = 2 Period = 2π Phase shift = 2 Vertical shift = up 1

$b = 1$

$\frac{c}{b} = 2$

$c = 2$

$y = 2\sin(\theta - 2) + 1$



Identify the amplitude, period, phase shift, vertical shift. Then sketch one period of the following:

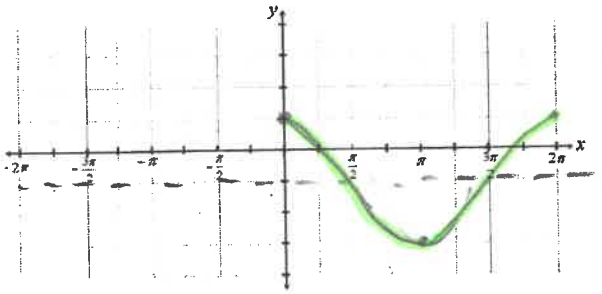
6). $y = 2\cos(x) - 1$

Amp = 2

Period = 2π

Ph Shift = 0

Vert Shift = ± 1



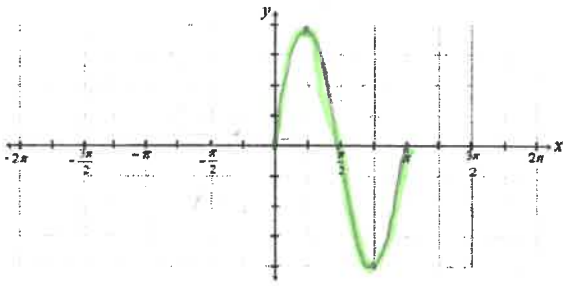
7). $y = 4\sin(2x)$

Amp = 4

Period = π

Ph Shift = 0

Vert Shift = 0



8). $y = 2\csc(2x)$ $\frac{1}{\sin}$ $2x = 0$ $2x = \pi$ $2x = 2\pi$

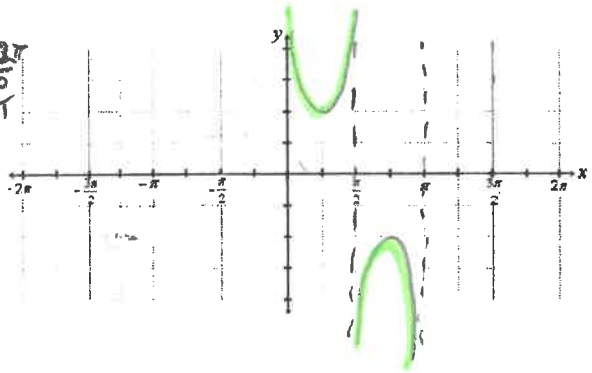
$x = 0$ $x = \frac{\pi}{2}$ $x = \pi$

Amp = 2

Period = π

Ph Shift = 0

Vert Shift = 0



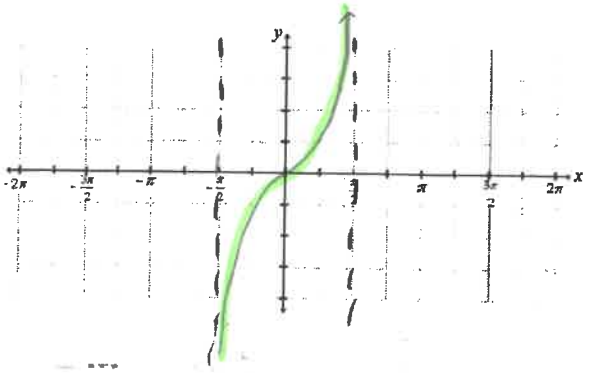
9. $y = \tan(x)$ $\frac{\sin}{\cos}$

Amp = —

Period = π

Ph Shift = 0

Vert Shift = 0



(40) For each angle given, i) sketch in standard position ii) give the quadrant where the angle lies, iii) List one positive and one negative coterminal angle.

a) $\frac{4\pi}{3}$

i) 

ii) III

iii) $\frac{-2\pi}{3}$, $\frac{10\pi}{3}$

b) $-\frac{5\pi}{6}$

i) 

ii) III

iii) $\frac{7\pi}{6}$, $\frac{-17\pi}{6}$

c) 210°

i) 

ii) III

iii) -150°, 570°

d) -135°

i) 

ii) III

iii) 225°, -495°

(41) Convert the angle measure from radians to degrees or degrees to radians.

a) 415°

$\frac{\pi}{180} = \frac{x}{415}$

$\frac{415\pi}{180} = x$

$\frac{83\pi}{36}$

b) -72°

$\frac{\pi}{180} = \frac{x}{-72}$

$\frac{-72\pi}{180} = x$

$\frac{-8\pi}{20} = x = \frac{-2\pi}{5}$

c) $\frac{5\pi}{7} \cdot \frac{5 \cdot 180}{7}$

$\approx 128.57^\circ$

d) $-\frac{3\pi}{5} = -108^\circ$

(42) Find the radian measure of the central angle of a circle with a radius of 12 ft that intercepts an arc of 25 ft.

$S = r\theta$

$25 = 12\theta \rightarrow \frac{25}{12} = \theta$

↑ radians

(43) Find the length of the arc on a circle with a radius of 20 m, intercepted by a central angle of 138°.

↑

$\frac{\pi}{180} = \frac{x}{138}$

$\frac{138\pi}{180} = x$

$S = r\theta$

$S = (20)\left(\frac{138\pi}{180}\right) = \frac{2760\pi}{180} \approx 15.33\pi$

≈ 48.17 m

(44) The radius of a CD is 6 cm. Find the linear speed of a point on the circumference of the disc if it's rotating at a speed of 500 revolutions per minute.

Linear speed $= \frac{s}{t} = \frac{r\theta}{t}$

$r = 6$ cm

$\theta = 2\pi \cdot 500$

$t = 1$ minute

Linear speed $= \frac{6(2\pi \cdot 500)}{1} = 6000\pi \frac{\text{cm}}{\text{min}} \approx 18849.555 \frac{\text{cm}}{\text{min}}$

(45) Evaluate the six trig. functions for the following angles.

a) $\frac{7\pi}{6}$

$\sin\frac{7\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

$\sec\frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$

$\csc\frac{7\pi}{6} = -2$

$\cot\frac{7\pi}{6} = 3/\sqrt{3}$

b) $\frac{3\pi}{4}$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{3\pi}{4} = -1$

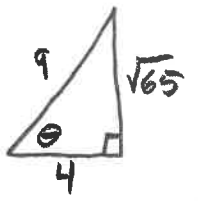
$\sec\frac{3\pi}{4} = -\frac{2}{\sqrt{2}}$

$\csc\frac{3\pi}{4} = \frac{2}{\sqrt{2}}$

$\cot\frac{3\pi}{4} = -1$

(46) Find the exact values of the six trig. functions for θ

a)



$\sin\theta = \frac{\sqrt{65}}{9}$

$\cos\theta = 4/9$

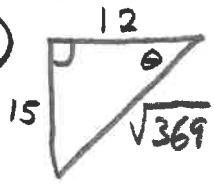
$\tan\theta = \frac{\sqrt{65}}{4}$

$\sec\theta = 9/4$

$\csc\theta = 9/\sqrt{65}$

$\cot\theta = 4/\sqrt{65}$

b)



$\sin\theta = 15/\sqrt{369}$

$\cos\theta = 12/\sqrt{369}$

$\tan\theta = 15/12$

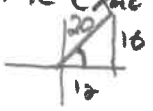
$\sec\theta = \sqrt{369}/12$

$\csc\theta = \frac{\sqrt{369}}{15}$

$\cot\theta = 12/15$

(47) Use the point from the terminal side of an angle in standard position, to determine the exact values of the six trig. functions

a) (12, 16)

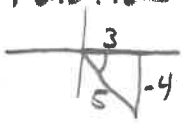


$\sin\theta = \frac{16}{20}$ $\quad \csc\theta = 20/16$

$\cos\theta = 12/20$ $\quad \sec\theta = 20/12$

$\tan\theta = \frac{16}{12}$ $\quad \cot\theta = 12/16$

b) (3, -4)



$\sin\theta = \frac{-4}{5}$ $\quad \csc\theta = 5/-4$

$\cos\theta = \frac{3}{5}$ $\quad \sec\theta = 5/3$

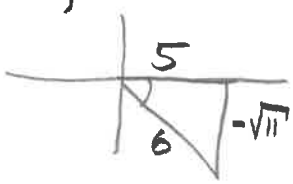
$\tan\theta = \frac{-4}{3}$ $\quad \cot\theta = -3/4$

(48) Find the values of the other 5 trig functions of θ if

$\sec\theta = \frac{6}{5}$, $\tan\theta < 0$

$\frac{1}{\cos\theta} = \frac{6}{5}$

$\cos\theta = \frac{5}{6}$



$\sin\theta = \frac{-\sqrt{11}}{6}$ $\quad \csc\theta = \frac{6}{-\sqrt{11}}$

$\cos\theta = \frac{5}{6}$

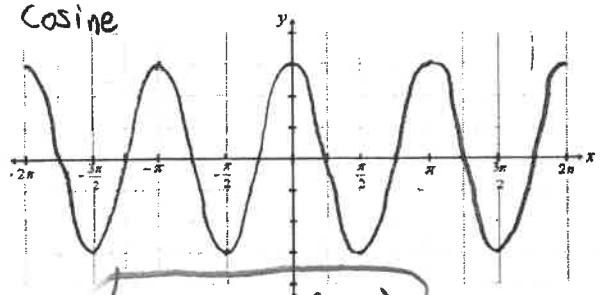
$\tan\theta = \frac{-\sqrt{11}}{5}$ $\quad \cot\theta = \frac{-5}{\sqrt{11}}$

(49) Evaluate without a calculator.

a) $\sin\frac{11\pi}{6} = -\frac{1}{2}$ b) $\cos 240^\circ$ $-\frac{1}{2}$ c) $\tan 4\pi = 0$

(50) Find an equation for the graphs

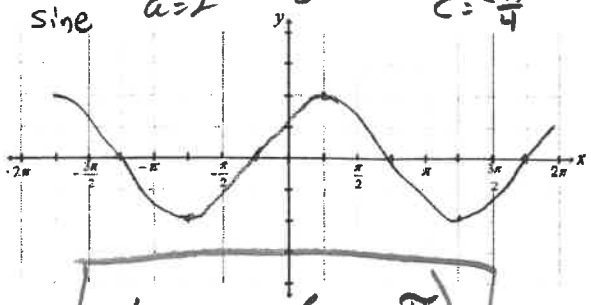
a) Cosine



$a=3$, $Per=\pi$, $b=2$, $P.S.=0$, $c=0$, $d=0$

$y = 3\cos(2x)$

b) Sine amp: 2, Per: 2π, P.S.: $\frac{-\pi}{4}$, d: 0, $a=2$, $b=1$, $c=\frac{-\pi}{4}$



$y = 2\sin\left(x + \frac{\pi}{4}\right)$

(51) Find exact values

a) $\sin^{-1}(-1) = -\frac{\pi}{2}$ b) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ c) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

d) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ e) $\tan^{-1}(1) = \frac{\pi}{4}$ f) $\tan^{-1}(0) = 0$

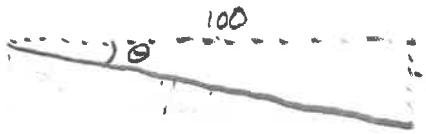
g) $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{\sqrt{2}}{2}$ ($\sin\frac{\pi}{4}$)

h) $\tan^{-1}\left(\cos\frac{\pi}{4}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) \approx 35.26^\circ$

i) $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{4}{3}$

(52) A road sign indicates that for next 4 miles the grade is 12%. Find the angle of descent and the change in elevation for a car descending down the mountain.

$\tan\theta = \frac{-12}{100}$



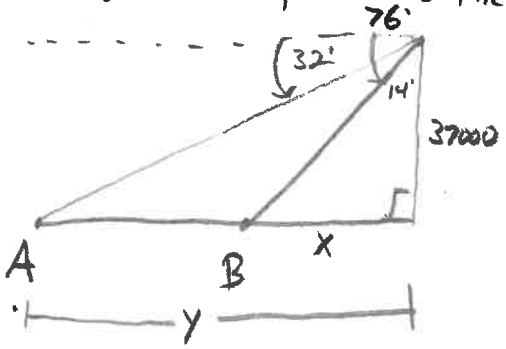
$\theta = \tan^{-1}\left(\frac{-12}{100}\right)$

$\theta \approx 6.84^\circ$

$\sin 6.84 = \frac{x}{4}$

$x \approx .4765$ miles

(53) A passenger in an airplane flying at an altitude of 37,000 ft sees two towns directly to the west. The angles of depression to the towns are 32° and 76°. How far apart are the two towns?



$\tan 14^\circ = \frac{x}{37000}$ $x = 9225.136$ ft

$\tan 58^\circ = \frac{y}{37000}$ $y = 59212.377$ ft

$A \to B = 59212.377 - 9225.136 = 49987.24$ ft