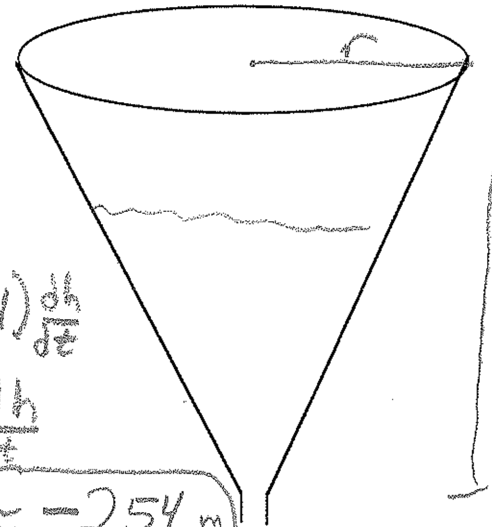
Calculus - Related Rates #4

Key

Example 10) Water is draining from a conical tank at the rate of 2 $\text{meter}^3/\text{sec}$. The tank is 16 meters high and its top radius is 4 meters. How fast is the water level falling when the water level is a) 12 meters high, b) 2 meters high?

$\frac{dV}{dt} = -2 \frac{m^3}{sec}$ $\frac{dh}{dt} = ?$



$h = 4r$
$r = \frac{1}{4}h$

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi \frac{1}{16} h^3 \Rightarrow V = \frac{1}{48}\pi h^3$

$\frac{dV}{dt} = \frac{1}{16}\pi h^2 \frac{dh}{dt}$

a) $-2 = \frac{1}{16}\pi(144)\frac{dh}{dt}$

$-2 = 9\pi \frac{dh}{dt}$

$\boxed{\frac{-2}{9\pi} = \frac{dh}{dt} \approx -.07 \frac{m}{sec}}$

b) $-2 = \frac{1}{16}\pi(4)\frac{dh}{dt}$

$-2 = \frac{1}{4}\pi \frac{dh}{dt}$

$\boxed{\frac{-8}{\pi} = \frac{dh}{dt} \approx -2.54 \frac{m}{s}}$

2. A sphere has a radius of 9 feet which is changing. Write formulas for its volume and surface area.

Volume	*Change of volume*	*Surface area*	*Change of surface area*
$V = \frac{4}{3}\pi r^3$	$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	$S = 4\pi r^2$	$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

a) its diameter is growing at the rate of 1 yd/min. (3 ft/min)

$r = 9'$ $\frac{dr}{dt} = 1.5 \frac{ft}{min}$

Change of volume

$\frac{dV}{dt} = 4\pi(9^2)(1.5)$

$\boxed{= 486\pi \quad \frac{dV}{dt} \approx 1526.8 \frac{ft^3}{min}}$

Change of surface area

$\frac{dS}{dt} = 8\pi(9)(1.5)$

$\boxed{= 108\pi \quad \frac{dS}{dt} \approx 339.29 \frac{ft^2}{min}}$

b) its radius is shrinking at the rate of $\frac{3}{4}$ inch/sec.

$\frac{dr}{dt} = -\frac{3}{4} \frac{in}{sec}$ $r = 108''$

Change of volume

$\frac{dV}{dt} = 4\pi(108^2)(-\frac{3}{4})$

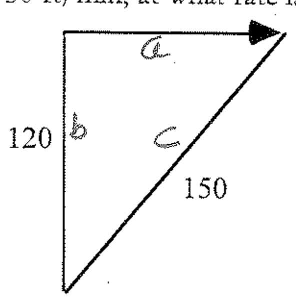
$\boxed{= -34992\pi \frac{in^3}{sec} \quad \frac{dV}{dt} \approx -109930.61 \frac{in^3}{sec}}$

Change of surface area

$\frac{dS}{dt} = 8\pi(108)(-\frac{3}{4})$

$\boxed{= -648\pi \frac{in^2}{sec} \quad \frac{dS}{dt} \approx -2035.75 \frac{in^2}{sec}}$

9. A boy flies a kite which is 120 ft directly above his hand. If the wind carries the kite horizontally at the rate of 30 ft/min, at what rate is the string being pulled out when the length of the string is 150 ft?



$a^2 + 120^2 = 150^2$

$a = 90$

$a^2 + b^2 = c^2$

$a^2 + 120^2 = c^2$

$2a\frac{da}{dt} = 2c\frac{dc}{dt}$

$2(90)(30) = 2(150)\frac{dc}{dt}$

$5400 = 300\frac{dc}{dt}$

$18 \frac{ft}{min} = \frac{dc}{dt}$

$\frac{da}{dt} = 30 \frac{ft}{min}$

$\frac{dc}{dt} = ?$

$c = 150'$

15. How fast does the water level drop when a cylindrical tank of radius 6 feet is drained at the rate of 3 ft^3/min?

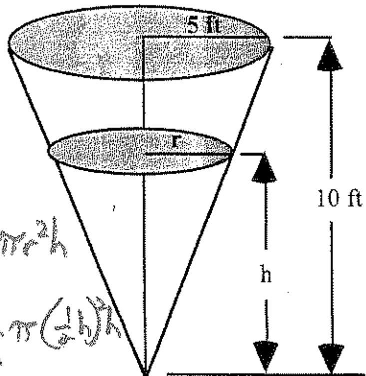
$\frac{dV}{dt} = -3 \frac{ft^3}{min}$ $r = 6'$ $\frac{dh}{dt} = ?$

$V = \pi r^2 h$

$V = 36\pi h \rightarrow \frac{dV}{dt} = 36\pi \frac{dh}{dt} \rightarrow \boxed{\frac{-3}{36\pi} = \frac{dh}{dt} \approx -.0265 \frac{ft}{min}}$

17. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands vertex down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep? *(Hint: use the picture and the variables shown below)*

$\frac{dV}{dt} = 9 \frac{ft^3}{min}$ $\frac{dh}{dt} = ?$ $h = 6$



r = radius of water surface at time t

h = depth of water surface at time t

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$r = \frac{1}{2}h$

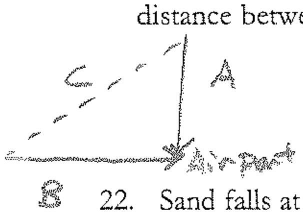
$2r = h$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$9 = \frac{1}{4}\pi(36)\frac{dh}{dt}$

$\boxed{\frac{9}{9\pi} = \frac{dh}{dt} = \frac{1}{\pi} \approx .3183 \frac{ft}{min}}$

19. Two commercial jets at 40,000 ft. are both flying at 520 mph towards an airport. Plane A is flying south and is 50 miles from the airport while Plane B is flying west and is 120 miles from the airport. How fast is the distance between the two planes changing at this time?



$A = 50$ miles $\frac{dA}{dt} = -520$ mph

$B = 120$ mile $\frac{dB}{dt} = -520$ mph

$C = 130$ $\frac{dC}{dt} = ?$

$A^2 + B^2 = C^2$

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$

$100(-520) + 240(-520) = 260\frac{dC}{dt}$

$\boxed{-680_{mph} = \frac{dC}{dt}}$

22. Sand falls at the rate of 30 ft^3/min onto the top of a conical pile. The height of the pile is always $\frac{3}{8}$ of the base diameter. How fast is the height changing when the pile is 12 ft. high?

$h = \frac{3}{8}d$

$h = \frac{3}{4}r \rightarrow r = \frac{4}{3}h$

$\frac{dh}{dt} = ?$ $h = 12$ $\frac{dV}{dt} = 30 \frac{ft^3}{min}$

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi\frac{16}{9}h^3 \rightarrow \frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$

$30 = \frac{16}{9}\pi(144)\frac{dh}{dt}$

$30 = 256\pi\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{30}{256\pi} \approx .0268 \frac{ft}{min}}$

24. A particle is moving along the curve whose equation is $\frac{xy^3}{1+y^2} = \frac{8}{5}$. Assume the x-coordinate is increasing at the rate of 6 units/sec when the particle is at the point (1, 2). At what rate is the y-coordinate of the point changing at that instant. Is it rising or falling?

$\frac{dx}{dt} = 6 \frac{u}{s}$ $x = 1$ $y = 2$ $\frac{dy}{dt} = ?$

cross multiply before derivative

$5xy^3 = 8 + 8y^2 \rightarrow 5y^3\frac{dx}{dt} + 15xy^2\frac{dy}{dt} = 16y\frac{dy}{dt}$

$5(8)(6) + 15(1)(4)\frac{dy}{dt} = 16(2)\frac{dy}{dt}$

$240 + 60\frac{dy}{dt} = 32\frac{dy}{dt}$

$240 = -28\frac{dy}{dt}$

$-\frac{240}{28} = \boxed{\frac{dy}{dt} = -\frac{60}{7}}$ $\boxed{\text{Falling}}$