Precalculus Exam Review
Semester 1

KEY

1.) What is the slope of the line that connects the points (-3, 7) and (9, -4)?

$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{7-(-4)}{-3-9} = \frac{11}{-12}$

2.) Write the equation of the line that contains the points (8, 3) and (0, -1).

$m = \frac{3-(-1)}{8-0} = \frac{4}{8} = \frac{1}{2}$

$-1 = \frac{1}{2}(0) + b$

$-1 = b$

$y = \frac{1}{2}x - 1$

3.) Write the equation of the line parallel to $y = 3x - 7$ that goes through (2, 8).

$m = 3$

$8 = 3(2) + b$

$8 = 6 + b$

$2 = b$

$y = 3x + 2$

4.) Let $f(x) = \frac{2x+3}{3-5x}$, find $f(2)$ and an expression for $f(x+1)$.

$f(2) = \frac{4+3}{3-10} = \frac{7}{-7} = -1$

$f(x+1) = \frac{2(x+1)+3}{3-5(x+1)} = \frac{2x+2+3}{3-5x-5} = \frac{2x+5}{-2-5x}$

5.) Identify the domain of the following functions.

a.) $f(x) = \sqrt{5x-21}$

$5x - 21 \geq 0$

$5x \geq 21$

$x \geq \frac{21}{5}$

b.) $f(x) = \frac{2x+3}{3-5x}$

$3 - 5x \neq 0$

$3 \neq 5x$

$\frac{3}{5} \neq x$

c.) $f(x) = \log_a(3x+9)$

$3x + 9 > 0$

$3x > -9$

$x > -3$

6.) Compared to $f(x) = x^3$, describe the transformations of $f(x) = (x-3)^3 + 5$.

shift right 3 shift up 5

7.) a.)Are these the graphs of functions? *Yes or No*

b.) Are the inverses of these graphs functions? *Yes or No*

i.) a) Not a function

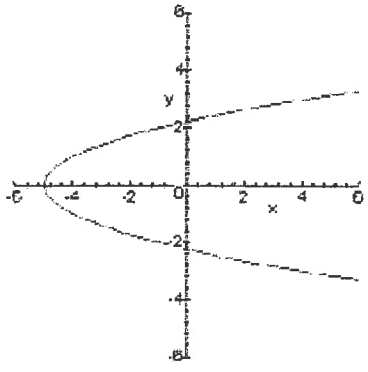
b) Yes inverse is a function

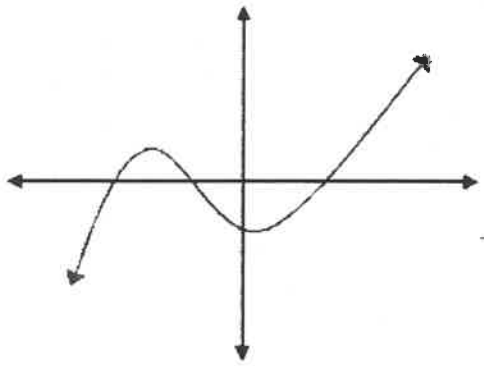
ii.) a) Function

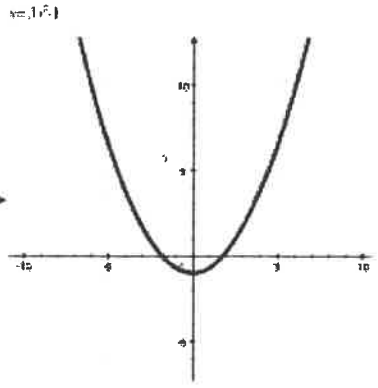
b) Inverse not a function

iii.) a) Function

b) Inverse not a function







8.) Let $f(x)=2x+7$ and $g(x)=5-6x$, find the value of the following.

a.) $(f+g)(2)=f(2)+g(2)$ b.) $\left(\frac{f}{g}\right)(-1)=\frac{f(-1)}{g(-1)}$ c.) $(g\circ f)(0)$

$f(2)=2(2)+7=11$
$g(2)=5-6(2)=-7$
$11+-7$
4

$\frac{5}{11}$

$g(f(0))$
$g(2(0)+7)$
$g(7)=5-6(7)$
-37

9.) What is the inverse of the function $f(x)=8x-15$?

$x=8y-15 \rightarrow x+15=8y \rightarrow \frac{x}{8}+\frac{15}{8}=y \rightarrow f^{-1}(x)=\frac{x}{8}+\frac{15}{8}$

10.) For the quadratic function, $f(x)=(x-5)^2-64$, identify the coordinates of the vertex and the x and y intercepts. Vertex: $(5,-64)$

x inter
$0=(x-5)^2-64$
$64=(x-5)^2$
$\pm 8=x-5$
$-8=x-5$ $x-5=8$
$-3=x$ $x=13$

y inter
$y=(0-5)^2-64$
$y=25-64$
$y=-39$

11.) How many real zeros **can** $f(x)=ax^8+bx^4-cx^3+d$ have?

8

12.) Identify the quotient, $q(x)$, and the remainder, $r(x)$, when $f(x)=4x^3+2x^2-13x+5$ is divided by $d(x)=x-6$.

6 | 4 2 -13 5
24 156 858
4 26 143 863

$\frac{143}{6}$
858

$q(x)=4x^2+26x+143$
$r(x)=863$

13.) Identify the other factors of $f(x)=x^3-2x^2-23x+60$ if $(x+5)$ is a factor.

-5 | 1 -2 -23 60
-5 35 -60
1 -7 12 0

$f(x)=(x+5)(x^2-7x+12)$
$=(x+5)(x-4)(x-3)$

14.) A degree 3 polynomial with real coefficients has zeros 5 and $9+4i$. What is the third zero?

$9-4i$

15.) Identify the horizontal and vertical asymptote(s) of $f(x)=\frac{3x^2+4x-1}{x^2-x-42}$.

No factor
$(x-7)(x+6)$

HA: $y=3$ (same degree ⇒ Leading coeff)
VA $x=7$ $x=-6$ (Factors of denom. only)

16.) Simplify $\sqrt{-289}$ into terms of the imaginary number i.

$\sqrt{-1}\cdot\sqrt{289} = i\,17 \rightarrow \boxed{17i}$

17.) Simplify: $\frac{2}{5+i}\,\frac{5-i}{5-i} = \frac{10-2i}{25+5i-5i-i^2} = \frac{10-2i}{25+1} = \frac{10-2i}{26} = \frac{10}{26} - \frac{2i}{26} = \boxed{\frac{5}{13} - \frac{i}{13}}$

18.) Write the logarithmic equation in exponential form or write the exponential equation in logarithmic form. a.) $\log_6 1296 = 4$ $\boxed{6^4 = 1296}$ b.) $5^3 = 125$ $\boxed{\log_5 125 = 3}$

19.) Condense the expression into the logarithm of a single quantity.

$(4\ln x + \ln y) - 2\ln z$

$\ln(x^4 y) - \ln z^2 \rightarrow \ln\left(\frac{x^4 y}{z^2}\right)$

20.) Expand the logarithm:

$\log\left(\frac{2x^4}{y}\right) = \log(2x^4) - \log y$

$\boxed{(\log 2 + 4\log x) - \log y}$

21.) Solve for x.

a.) $\log_4(x-6) = 4$ $4^4 = x-6 \rightarrow 256 = x-6$ $\boxed{262 = x}$

b.) $4^{3x} = 35$ $3x = \log_4 35 \rightarrow \frac{3x}{3} = \frac{2.5646}{3} \rightarrow \boxed{x = .85488}$

22.) Find the amount in an account that invested \$4200 at a rate of 6% over a period of 5 years if the interest is:

a.) compounded quarterly $A = 4200\left(1 + \frac{.06}{4}\right)^{4\cdot 5}$ $A = 5656.79$ $\boxed{\$5656.79}$

b.) compounded continuously $A = 4200e^{.06\cdot 5} = \boxed{\$5669.41}$

23.) Use the unit circle to evaluate the following.

a.) $\sin\frac{\pi}{6}$ $\boxed{\frac{1}{2}}$ b.) $\cos\frac{-7\pi}{6}$ $\boxed{\frac{-\sqrt{3}}{2}}$ c.) $\tan\frac{5\pi}{4}$ $\boxed{1}$

24.) Find the value of $\cos\theta$, given that $\sin\theta = \frac{24}{25}$ and θ is in quadrant II.

$\boxed{\cos\theta = \frac{-7}{25}}$

25.) Find the exact value of each expression.

a.) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b.) $\cos^{-1}\left(\frac{1}{2}\right)$ c.) ~~$\sin^{-1}\left(\sin\frac{7\pi}{4}\right)$~~

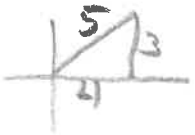
$\sin\theta = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ $\frac{\pi}{3}, \frac{5\pi}{3}$

26.) Identify the amplitude, period and ~~phase shift~~ for the equation $f(x) = 5\cos(8x)$.

Amp: 5 Per: $\frac{2\pi}{8} = \frac{\pi}{4}$

27.) Given that $\sin x = \frac{3}{5}$ find the other 5 trig functions. $0 < X < 90$



cos x = $\frac{4}{5}$

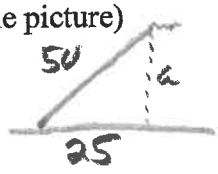
tan x = $\frac{3}{4}$

csc x = $\frac{5}{3}$

sec x = $\frac{5}{4}$

cot x = $\frac{4}{3}$

28.) A kite is flying on a string that is 50 feet long, the wind blows it so it is 25 feet downwind of the person holding the string. How high in the sky is the kite (to the nearest tenth of a foot)? (Draw the picture)



$a^2 + 25^2 = 50^2$

$a^2 = 1875$

$a = 43.3'$

29.) Find the angular velocity and linear velocity of a merry-go-round that is spinning at 5 revolutions per minute. The radius of the merry-go-round is 5 feet.

$\theta = 5 \cdot 2\pi = 10\pi$

$r = 5$

$\frac{5 \cdot 10\pi}{1\text{ min}} = \frac{50\pi}{1}$

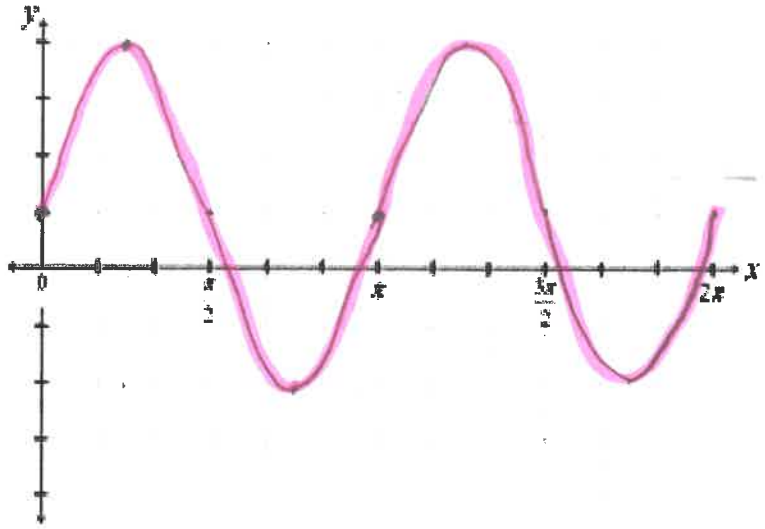
Angular velocity $\frac{10\pi \text{ rad}}{1 \text{ min}}$ radians per min

Linear velocity 50π feet per minute

30.) Graph the function: f(x) = 1 + 3sin(2x)

Include the Amplitude, Period, Vertical Shift, and ~~Phase Shift~~. Write 'none' for any that do not exist.

3 $\frac{2\pi}{2} = \pi$ up 1



Amplitude: 3

Period: π

Vertical Shift: up 1

~~Phase Shift:~~