Precalculus Trig. Review for Test

Key

Prove the Identity

① $\csc\theta \cdot \cos\theta = \cot\theta$

$\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1} =$

$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$ ■

② $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$\sin\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \frac{1}{\cos\theta}$

$\frac{\cos}{\cos}\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta} =$

$\frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} =$

$\frac{1}{\cos\theta} = \frac{1}{\cos\theta}$ ■

③ $\sec\theta - \cos\theta = \sin\theta\tan\theta$

$\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\left(\frac{\cos}{\cos}\right) = \frac{\sin\theta}{1} \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos} = \frac{\sin^2\theta}{\cos\theta}$

$\frac{1-\cos^2\theta}{\cos\theta} =$

$\frac{\sin^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$ ■

④ $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$(\sin\theta+\cos\theta)(\sin\theta+\cos\theta) + (\sin\theta-\cos\theta)(\sin\theta-\cos\theta) =$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta =$

$(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) =$

$1 + 0 + 1 =$

$2 = 2$ ■

⑤ $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$1 - \frac{1-\cos^2\theta}{1-\cos\theta} =$

$1 - \frac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{1-\cos\theta}} =$

$1 - (1+\cos\theta) =$

$1 - 1 - \cos\theta =$

$-\cos\theta = -\cos\theta$ ■

⑥ $\tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta + \frac{\cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta =$

$\sin^2\theta + \cos^2\theta =$

$1 = 1$ ■

⑦ $\frac{1+\sin\theta}{1-\sin\theta} = \frac{\csc\theta + 1}{\csc\theta - 1}$

$(1+\sin\theta)\left(\frac{1}{\sin\theta} - 1\right) = \left(\frac{1}{\sin\theta} + 1\right)(1-\sin\theta)$

$\frac{1}{\sin\theta} + 1 - 1 - \sin\theta = \frac{1}{\sin\theta} + 1 - 1 - \sin\theta$ ■

Solve the trig. equation for θ, $0 \le \theta < 2\pi$.

(8) $2\sin\theta + \sqrt{3} = 0$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\boxed{\theta = \frac{4\pi}{3}, \frac{5\pi}{3}}$

(9) $\sin(2\theta) = \frac{1}{2}$

$2\theta = \frac{\pi}{6}$ $\quad 2\theta = \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}$ $\quad \theta = \frac{5\pi}{12}$

(10) $\tan(\theta - \frac{\pi}{2}) = 1$

$\theta - \frac{\pi}{2} = \frac{\pi}{4}$ $\quad \theta - \frac{\pi}{2} = \frac{5\pi}{4}$

$\theta = \frac{3\pi}{4}$ $\quad \theta = \frac{7\pi}{4}$

(11) $3\sqrt{2}\cos\theta + 2 = -1$

$3\sqrt{2}\cos\theta = -3$

$\cos\theta = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \frac{-\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

(12) $5\csc\theta - 3 = 2$

$5\csc\theta = 5$

$\csc\theta = 1$

$\frac{1}{\sin\theta} = 1$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

(13) $2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ $\quad \sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad \theta = \frac{\pi}{2}$

(14) $3\cos\theta + 3 = 2\sin^2\theta$

$3\cos\theta + 3 = 2(1 - \cos^2\theta)$

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$2\cos\theta + 1 = 0$ $\quad \cos\theta + 1 = 0$

$\cos\theta = \frac{-1}{2}$ $\quad \cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ $\quad \theta = \frac{3\pi}{2}$

(15) $\cos^2\theta + \sin\theta = 2$

$1 - \sin^2\theta + \sin\theta - 2 = 0$

$-\sin^2\theta + \sin\theta - 1 = 0$

$0 = \sin^2\theta - \sin\theta + 1 = 0$

Doesn't Factor

Can't Solve $\Rightarrow$

$\boxed{\text{No Solution}}$

(16) $2\sin^2\theta = 3(1 - \cos\theta)$

$2(1 - \cos^2\theta) = 3 - 3\cos\theta$

$2 - 2\cos^2\theta = 3 - 3\cos\theta$

$0 = 2\cos^2\theta - 3\cos\theta + 1$

$0 = (2\cos\theta - 1)(\cos\theta - 1)$

$2\cos\theta - 1 = 0$ $\quad \cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = 1$

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \theta = 0}$

17) which quadrant does θ lie in if...

a) $\sin\theta > 0$ and $\cos\theta < 0$

I II   II III

II

b) $\sin\theta > 0$ and $\tan\theta > 0$

I II   I III

I

c) $\sec\theta > 0$ and $\tan\theta < 0$

I IV   II IV

IV

d) $\csc\theta < 0$ and $\cot\theta > 0$

III IV   I III

III

Solve each triangle. (If the Δ exists, or if two Δ's exist)

18) $\measuredangle A = 50°$, $\measuredangle B = 30°$, $a = 1$

$\measuredangle C = 100°$

$\frac{\sin 50}{1} = \frac{\sin 30}{b}$

$b = .6527$

$\frac{\sin 50}{1} = \frac{\sin 100}{c}$

$c = 1.286$

19) $a = 3$, $\measuredangle A = 10°$, $b = 4$

$\frac{\sin 10}{3} = \frac{\sin B}{4}$

$.2315 = \sin B$

Δ#1

$\measuredangle B = 13.38$

$\measuredangle C = 156.62$

$\frac{\sin 10}{3} = \frac{\sin 156.62}{c}$

$c = 6.86$

Δ#2

$\measuredangle B = 166.61$

$\measuredangle C = 3.39°$

$\frac{\sin 10}{3} = \frac{\sin 3.39}{c}$

$c = 1.02$

20) $a = 3$, $c = 1$, $\measuredangle B = 100°$

$b^2 = 3^2 + 1^2 - 2(3)(1)\cos 100$

$b^2 = 10 - 6\cos 100$

$b^2 = 11.0418$

$b = 3.32$

$\frac{\sin 100}{3.32} = \frac{\sin A}{3}$

$.889 = \sin A$

$\measuredangle A = 62.76°$

$\measuredangle C = 17.24°$

21) $a = 2$, $b = 3$, $c = 1$

$3^2 = 2^2 + 1^2 - 2(2)(1)\cos B$

$9 = 4 + 1 - 4\cos B$

$4 = -4\cos B$

$-1 = \cos B$

$\measuredangle B = 180°$

Not a Δ